An Optimization Approach to Variable Reluctance Resolver

Longfei Xiao, Zheng Li, and Chao Bi

School of Optical-Electrical and Computer Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China

An approach for optimizing the rotor shape of a variable reluctance (VR) resolver is presented. In the optimization, the shape of the rotor is described with Fourier series. The Taguchi optimization method, combined with a finite-element method (FEM), is used to calculate the value of the objective function of the VR resolver and make the error of the angular position of the VR resolver to be minimum. The VR resolver with three-pole pairs is used as an example to show the effectiveness of the optimization approach.

Index Terms—**Finite-element method (FEM), objective function, optimization, position error, Taguchi, variable reluctance (VR), VR resolver.**

I. Introduction

DETECTING the rotor position precisely is necessary for many applications, e.g., the servowriter in hard-disk drive (HDD). In order to get precise rotor-position signals, angle sensors are used logically.

There are many different types of angle sensors [1], such as the optical encoder, Hall sensor, and resolver. The optical encoders are widely used in industry for their high resolution. But this kind of sensor utilizes the local optical effect in angle detection and is sensitive to environmental conditions, e.g., vibration, smoking, and dust. The Hall sensor uses the local magnetic field generated by the permanent magnet, and its resolution is low, as generating the required magnetic-field waveform precisely is difficult. All of these limit the application of the sensors that utilize the local effect to detect the angle position. One example is an electric vehicle (EV) because the vibration in EV could be very strong at high speed [2]. The servowriter used in HDD production is another example, as its rotational speed is high and requires precise angle in its operation.

By contrast, variable reluctance (VR) resolvers utilize the global effects of the electromagnetic (EM) system and are not sensitive to the environmental condition. This kind of sensor is thus most suitable for the applications, such as EV and heavy machines [3], [4], because of its higher temperature resistance, anti-impact ability, smaller dimensions, and so on. This kind of angle sensor should also be suitable for the precise equipment, e.g., the servowriter used in HDD production.

Compared with other sensors, the structure of the VR resolver is simple [2]–[4]. However, there are still some difficulties in the design of rotor shape and winding distributions, which are important factors affecting the accuracy of output signals [5], [6]. For reducing the signal error, many studies concentrate on winding distributions and signal processing [7], [8]. The design of the rotor shape is less involved, nor is there detailed description of the rotor shape. Like its complicated EM structure, the optimization of the rotor shape of the VR resolver is certainly very difficult.

Manuscript received April 14, 2019; revised September 9, 2019 and October 12, 2019; accepted November 5, 2019. Date of current version January 20, 2020. Corresponding author: L. Xiao (e-mail: long.fei.xiao@foxmail.com).

Color versions of one or more of the figures in this article are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TMAG.2019.2953255

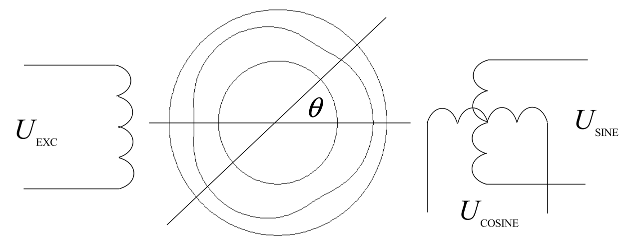

Fig. 1. Basic operating principle of the VR resolver.

For realizing an effective optimization of the VR resolver, the authors formulated the rotor shape with Fourier series. Furthermore, a finite-element method (FEM) is used to generate the samples of the objective function, and the Taguchi method is adopted to optimize the VR resolver.

II. Design of Rotor Shape Based on Fourier Series

The basic operating principle of the VR resolver is specifically introduced in [7]–[9]. The salient pole effect is the key to generating the rotor position signal. As shown in Fig. 1, the variation of air-gap permeance due to the rotation of the salient pole rotor changes the mutual inductance between the two-phase output windings that are electrically orthogonal to each other. Then, the SINE and COSINE output signals are expected to be generated. Ultimately, the position of the rotor can be obtained by decoding the output signals through the resolver-to-digital converter (RDC) [10], [11].

The structure of the 3-X VR resolver is shown in Fig. 2. It should be noted that the exciting and output windings with alternate polarities are all arranged in the stator. The number of turns of the exciting winding on each tooth is 30 and that of the output winding is 120, as shown in Fig. 2(c).

The relationship between the stator teeth and rotor poles can be expressed by [12]

$$z = 2mp \tag{1}$$

where z is the number of stator teeth; and m and p represent the phases of output windings and the pole pairs of the rotor, respectively.

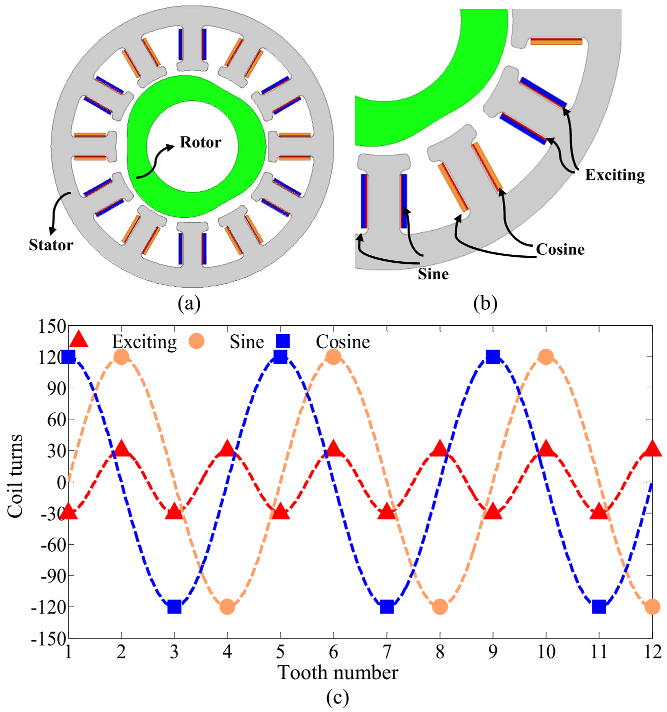

Fig. 2. 3-X VR resolver with 12 stator slots. (a) Stator and rotor. (b) Local enlarged drawing. (c) Winding distributions.

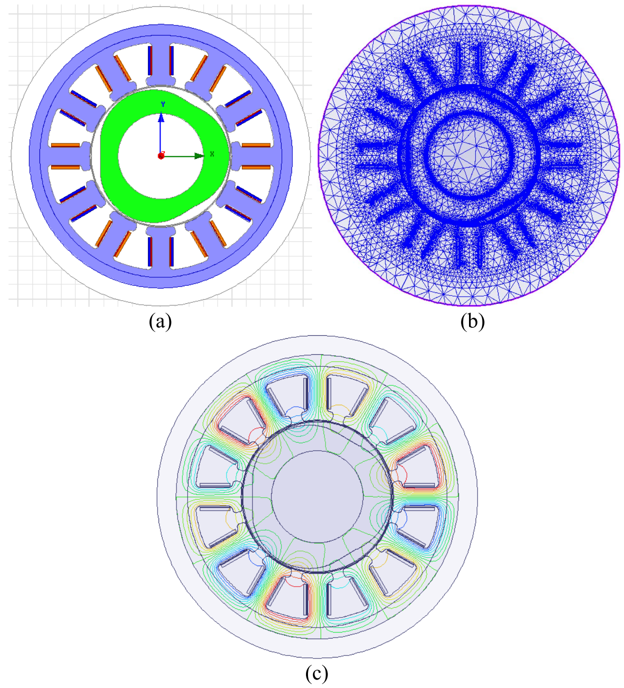

Fig. 3. FEM model of the 3-X VR resolver. (a) 2-D simulation model. (b) Finite-element meshing diagram. (c) Flux distribution.

In [3], [9], and [13], different analysis methods for the VR-resolver design are presented, which include a magnetic-circuit method and a finite-element analysis (FEA), and a great emphasis was placed on the calculation of air-gap permeance. Then, the rotor shape is designed according to the results of FEA. But it is difficult and time consuming to calculate the air-gap permeance of the VR resolver by the existing methods, as the EM structure of the resolver is quite complex. Moreover, the initial design is not easy to optimize.

It is clear that the accuracy of the output signal of the output windings is affected by many factors of the VR resolver, and one of them is the rotor shape. The fundamental requirements of the resolver include that its SINE and COSINE windings should be able to generate sinusoidal envelope of output signals to the rotor position, and these two signals should be orthogonal in the space domain. Ge and Zhu [6] proposed an air-gap length function for the ability of the resolver. However, this formula has only two form factors for optimization, and it is not enough to reduce the error of the rotor position signal.

As the periodicity of the EM structure shown in Fig. 2, the rotor shape can be described by Fourier series

$$\begin{cases} r(\theta) = h_0 + \sum_{i=1}^{\infty} h_i \cos(ip\theta + \varphi_i) \\ h_0 = (1/\theta_1) \int_{\theta_0}^{\theta_0+\theta_1} r(\theta) d\theta \\ h_i = \sqrt{a_i^2 + b_i^2} \\ a_i = (2/\theta_1) \int_{\theta_0}^{\theta_0+\theta_1} r(\theta) \cos(ip\theta) d\theta \\ b_i = (2/\theta_1) \int_{\theta_0}^{\theta_0+\theta_1} r(\theta) \sin(ip\theta) d\theta \\ \varphi_i = -\arctan(b_i/a_i) \\ \theta_1 = 2\pi/p \end{cases} \tag{2}$$

where r is the rotor radius; θ is the angle of rotor position; and h_i and h_0 are the form factors of the rotor shape.

It is obvious that the Fourier series can describe all of the periodic functions, which include the rotor shape as the shape function of the rotor surface is periodic. In this article, (2) is used to describe this function.

Modifying the rotor shape can be processed by adjusting the shape factors, i.e., amplitudes of the harmonics of the shape. That is, the optimization of the rotor shape means also looking for the suitable amplitudes of the shape harmonics. Combined with the Taguchi method, such an optimization process should be simple and effective.

III. Optimization of Rotor Shape

A. FEM Model

In this article, for realizing the effective optimization, an FEM model of the 3-X VR resolver is presented, as shown in Fig. 3. For simplifying the calculation procedure, the end turn effects of the winding are ignored. In the calculation, the rotor speed is set at 80 rpm, and the frequency of the excitation current is 400 Hz, i.e., the exciting current can be expressed by

$$i_e(t) = 0.032 \sin(2\pi f t) = 0.032 \sin(800\pi t). \tag{3}$$

The air-gap field can affect the performance of the resolver directly. In order to improve the accuracy of FEM, more detailed dissections are used to describe the air-gap region, as shown in Fig. 3(b). The flux distribution obtained with FEM is shown in Fig. 3(c). Thus, it is impossible for the designers to accurately calculate the air-gap permeance and output voltage by a mathematical method.

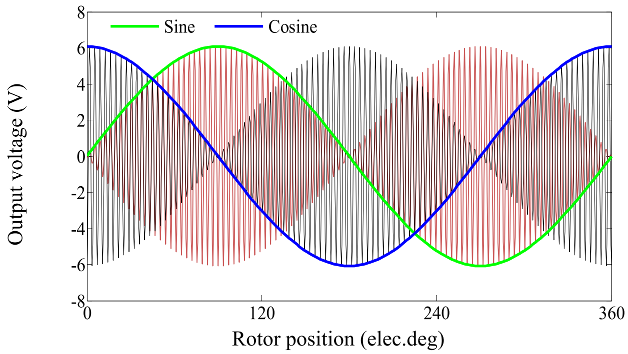

Fig. 4. Output voltage obtained by FEM.

Fig. 5. Prototype of the VR resolver rotor optimized.

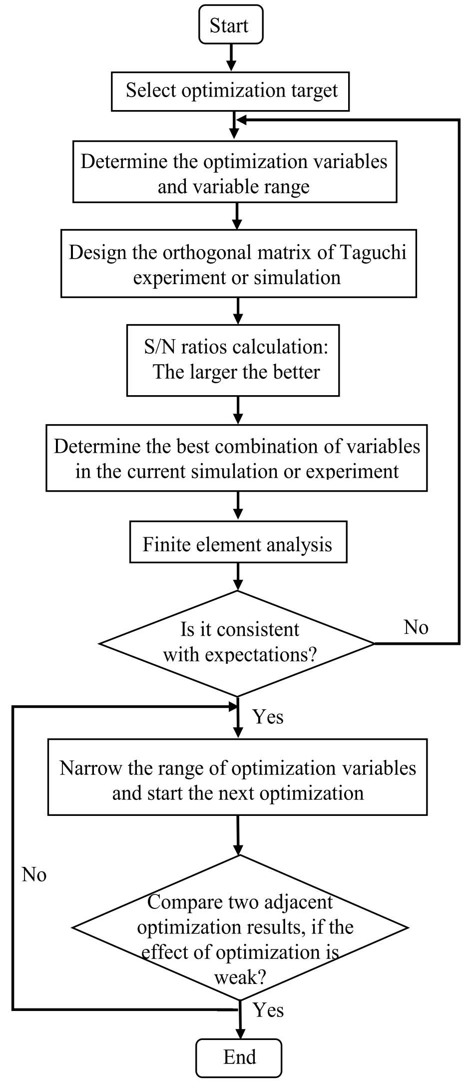

Fig. 6. Basic process of the Taguchi optimization method.

The output signals of the SINE and COSINE windings are analog values, as shown in Fig. 4. The periodic number of the signals is equal to the number of rotor pole-pairs.

The prototype of the rotor optimized is shown in Fig. 5. Due to page length limitations, the testing results of the resolver cannot be presented in this article, and the authors will introduce the testing procedure and the optimization results of the VR resolver in another article.

B. Taguchi Optimization Method

The Taguchi optimization method was presented in the 1950s by Gen'ichi Taguchi. These days, this optimization method has been widely used in many fields, e.g., motors, automobiles, and chemicals. Compared with the other design of experiments (DOE), the main advantage of the Taguchi method is that it can use the discrete results to explore the optimal point [14]–[16]. The main tools of the Taguchi method are orthogonal table and signal-to-noise ratio (S/N ratio). The former is utilized to reduce the number of experiments or simulations. The latter is adopted to select the best combination of factor level and determine the next optimization.

The process of the Taguchi optimization method is shown in Fig. 6. In this section, there are two objective functions. One is total harmonic distortion (THD), which is expressed by

$$\text{THD} = \sqrt{\sum_{n=2}^{\infty} V_n^2} \Big/ V_1 \tag{4}$$

where V_1 is the amplitude of the fundamental component of output voltage and V_n is the amplitude of the nth other harmonics.

Another objective function is the electrical angle error of the rotor, and it can be expressed by

$$\text{Position error} = \max|\text{error}(i)| \tag{5}$$

where error(i) represents the electrical angle error between the calculated value and the ideal value.

The optimization variables are shape factors mentioned in Section II. A number of orthogonal arrays have been proposed by Gen'ichi Taguchi. The orthogonal matrix, which is used in this paper, is shown in Table I.

TABLE I
SIMULATION RESULTS OF THE FIRST STAGE

No	Factor A	Factor B	Factor C	Factor D	THD (%)	Position error (elec.deg)	S/N
1	1	1	1	1	0.305	0.0354	29.014
2	1	2	2	2	0.428	0.1251	18.052
3	1	3	3	3	0.556	0.2165	13.293
4	2	1	2	3	0.611	0.2249	12.959
5	2	2	3	1	0.778	0.3129	10.091
6	2	3	1	2	0.860	0.3794	8.4180
7	3	1	3	2	0.979	0.4268	7.3952
8	3	2	1	3	1.072	0.4949	6.1101
9	3	3	2	1	1.233	0.5783	4.7565

TABLE II
SIMULATION RESULTS OF THE SECOND STAGE

No	Factor A	Factor B	Factor C	Factor D	THD (%)	Position error (elec.deg)	S/N
1	1	1	1	1	0.196	0.1214	18.313
2	1	2	2	2	0.195	0.0748	22.524
3	1	3	3	3	0.222	0.0314	30.060
4	2	1	2	3	0.259	0.0253	31.950
5	2	2	3	1	0.317	0.0385	28.289
6	2	3	1	2	0.350	0.0716	22.899
7	3	1	3	2	0.405	0.0866	21.250
8	3	2	1	3	0.445	0.1221	18.266
9	3	3	2	1	0.527	0.1677	15.508

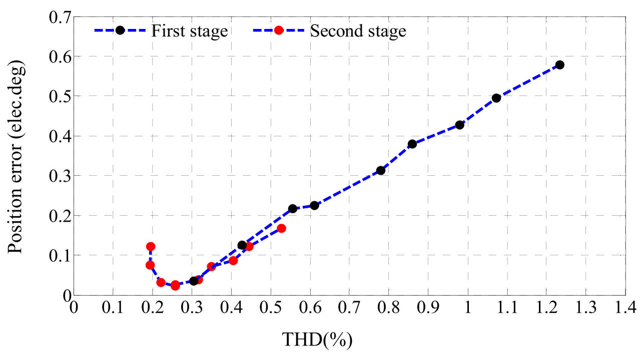

Fig. 7. Relationship between THD and position error.

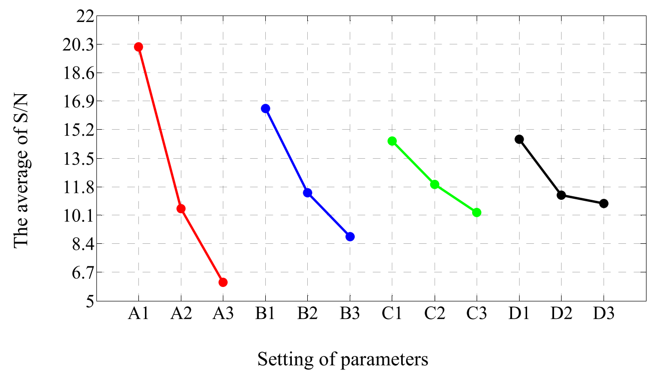

Fig. 8. Main effect diagram of S/N in the first stage.

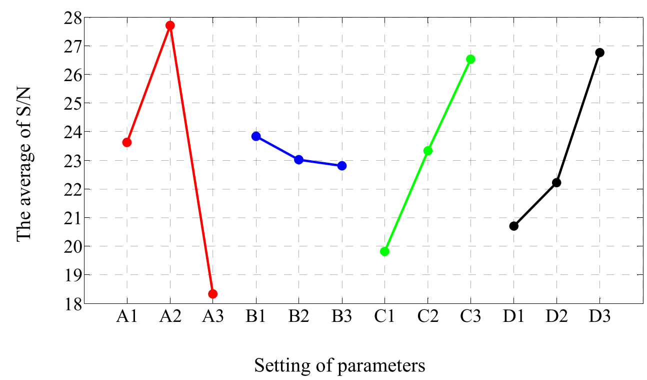

Fig. 9. Main effect diagram of S/N in the second stage.

For calculating the S/N ratio, three formulas are the ones most widely used

$$\text{Nominal is Better: } SN_N = 10 \log\left(\bar{y}^2/s^2\right) \tag{6}$$

$$\text{Larger is Better: } SN_L = -10 \log\left[\left(\sum_{i=1}^{n} 1/y_i^2\right)\Big/ n\right] \tag{7}$$

$$\text{Smaller is Better: } SN_S = -10 \log\left[\left(\sum_{i=1}^{n} y_i^2 \Big/ n\right)\right] \tag{8}$$

where $\bar{y}$ and s represent the average value and the standard deviation of objective functions, respectively.

Equation (8) is adopted in this section. It should be noted that the values of "THD and position" should be minimum, but the S/N ratio should be maximum.

The simulation results of the first stage are shown in Table I, where $A = h_1, B = h_2, C = h_4$, and $D = h_5$.

In the first stage, level 2 is the initial level, and levels 1 and 3 are 1.05 and 0.95 times of the initial level, respectively. As shown in Fig. 7 and Table I, there is a clear trend that when these four shape factors are closer to level 1, S/N increases with the decrease of THD and position error. Actually, an intuitive optimization direction is indicated in this trend. The impact of optimization variables on the optimization target is reflected in Fig. 8. It is obvious that the influence of A, B, C, and D is, in turn, weakened. According to the principle that the S/N ratio should be maximum, the best combination of factor level of the first stage is A1B1C1D1.

In the second stage, level 2 is the best factor level of the first stage. Levels 1 and 3 represent 1.025 and 0.975 times of level 2, respectively. The simulation results of the second stage are shown in Table II.

Fig. 9 reflects the best combination of factor level of the second stage (A2B1C3D3). And it should be noted that the impact proportion of shape factors on the optimization target is not immutable. Compared with the first stage, the effect of B becomes weak. By contrast, the impact of C and D is improved.

Comparing the results shown in Fig. 10, it can be found that the position error declines from 0.3037° to 0.0220°, and the THD is also reduced from 0.755% to 0.258%. The effect of the optimization is obvious and significant in the 3-X VR resolver design.

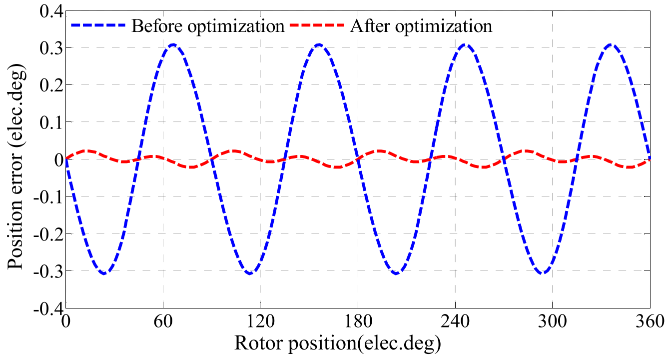

Fig. 10. Position errors of the 3-X VR resolver in FEM.

IV. Conclusion

As the complex EM structure of the VR resolver, FEM is an effective tool to describe the structure and verify the performance of the VR resolver designed. The values of the objective function of the optimization on the VR resolver are thus discrete. For this kind of optimization, traditional optimization based on an analytical model is disabled. The Taguchi optimization method is an effective tool in exploring the best point of the optimization with discrete objective function values and, thus, is suitable to the cases of EM design with FEM. In our practice, the time spent on simulations or experiments can be drastically reduced, and the design of the parameter level becomes relatively simple by using the Taguchi method.

When the rotor shape is described by the Fourier series, modifying the rotor shape can be processed by adjusting the amplitudes of the harmonics of the shape. This article has shown the effectiveness of using such a structure module.

The results presented in this article confirmed the effectiveness of the optimization with the combination of the FEM and Taguchi method in the VR resolver design. It is clear that the optimization approach presented in this article can also be used in the design of other types of resolvers and even in the design of other EM systems of electric machines.

References

[1] R. Setbaken, "System performance and application tradeoffs determine the choice between encoders and resolvers in brushless servos," *Power Convers. Intell. Motion*, vol. 22, no. 5, pp. 69–76, 1996.

[2] L. Sun, J. Shang, and J. Zou, "New absolute rotor-position sensors for inverter-driven motors," in *Proc. Int. Magn. Conf. (INTERMAG)*, Apr. 2005, pp. 975–976.

[3] L. Sun, "Analysis and improvement on the structure of variable reluctance resolvers," *IEEE Trans. Magn.*, vol. 44, no. 8, pp. 2002–2008, Aug. 2008.

[4] K.-C. Kim, C. S. Jin, and J. Lee, "Magnetic shield design between interior permanent magnet synchronous motor and sensor for hybrid electric vehicle," *IEEE Trans. Magn.*, vol. 45, no. 6, pp. 2835–2838, Jun. 2009.

[5] C.-S. Jin, I.-S. Jang, J.-N. Bae, J. Lee, and W.-H. Kim, "Proposal of improved winding method for VR resolver," *IEEE Trans. Magn.*, vol. 51, no. 3, Mar. 2015, Art. no. 8102404.

[6] X. Ge and Z. Q. Zhu, "A novel design of rotor contour for variable reluctance resolver by injecting auxiliary air-gap permeance harmonics," *IEEE Trans. Energy Convers.*, vol. 31, no. 1, pp. 345–353, Mar. 2016.

[7] R. Alipour-Sarabi, Z. Nasiri-Gheidari, H. Oraee, and F. Tootoonchian, "Performance analysis of concentrated wound-rotor resolver for its applications in high pole number permanent magnet motors," *IEEE Sensors J.*, vol. 17, no. 23, pp. 7877–7885, Jan. 2017.

[8] A. Murray, B. Hare, and A. Hirao, "Resolver position sensing system with integrated fault detection for automotive applications," in *Proc. IEEE Sensors*, vol. 2, Jun. 2002, pp. 864–869.

[9] S. Cui and H. Ge, "Stator structure design and analysis of variable reluctance resolver for hybrid-vehicle motor drive," in *Proc. 7th Int. Power Electron. Motion Control Conf.*, Jun. 2012, pp. 2587–2592.

[10] N. Ismail, F. Kobayashi, and M. Inoue, "Integrally accurate resolver-to-digital converter (RDC)," in *Proc. 10th Asian Control Conf. (ASCC)*, May 2015, pp. 1–4.

[11] L. Shao, Z. Tang, K. Maki, H. Funato, J. Moore, and G. Saikalis, "Integrated simulation and analysis of resolver sub-system for HEV electric drive," in *Proc. IEEE Vehicle Power Propuls. Conf.*, Sep. 2008, pp. 1–5.

[12] A. Nagarkatti, R. E. Lordo, and B. Kind, "Harmonically graded airgap reluctance-type rotating electric resolver," U.S. Patent 4 631 510 A, Dec. 23, 1986.

[13] D. C. Hanselman, R. E. Thibodeau, and D. J. Smith, "Variable-reluctance resolver design guidelines," in *Proc. 15th Annu. Conf. IEEE Ind. Electron. Soc.*, Nov. 1989, pp. 203–208.

[14] I. Arizono, A. Kanagawa, H. Ohta, K. Watakabe, and K. Tateishi, "Variable sampling plans for normal distribution indexed by Taguchi's loss function," (in Japanese), *Naval Res. Logistic*, vol. 44, no. 6, pp. 591–603, 1997.

[15] S. Fujikawa, "Optimum parameter design using the Taguchi method for Finite-element analysis of 3D forging deformation," (in Japanese), *J. Jpn. Soc. Technol., Plasticity*, vol. 40, no. 466, pp. 1061–1065, 1999.

[16] I. Tanabe, "Development of a tool for the easy determination of control factor interaction in the design of experiments and the taguchi methods," in *Proc. Int. Conf. Control, Artif. Intell., Robot. Optim. (ICCAIRO)*, May 2017, pp. 301–306.