XIX International Conference on Electrical Machines - ICEM 2010, Rome

An Optimal Motor Drive Current Model and Its Application to Reluctance Motor

C. BI, N. P. Hla, C.S. Soh, Q. Jiang

***Abstract* -- Based on the analysis of the motor parameters, an optimal drive current model of synchronous motor is derived with constrained optimal analysis. The results show that, if the inductances of the synchronous motor can be obtained accurately in the space domain, optimal current can be obtained which generates the required electromagnetic torque with minimum copper loss. Analysis elucidates the influence of the high order inductance harmonics to the torque ripple and the waveform of the optimal current. These harmonics cause the optimal drive current to be non-sinusoidal. However, the analysis proves that, in the idealized state, the optimal current is sinusoidal, and the optimal phase angle is determined by the motor inductances. The application of the model to reluctance motor is presented in the paper, and the results confirm the effectiveness of the presented current model and analysis method.**

***Index Terms*-- Optimal control, optimization method, reluctance motor drives, synchronous motor drives, torque control.**

I. INTRODUCTION

In this paper, the optimal drive current of synchronous motor is defined as the current which can generate the required constant electromagnetic (EM) torque with minimum copper loss. In the analysis, the motor parameters are independent to the drive currents.

It is clear that the optimal drive current defined is expected to drive synchronous motors in all applications. Sinusoidal waveform has been accepted widely as the optimal current waveform in many applications. However, this paper will show that, when the high order harmonics of the winding inductances are included, the optimal drive current is not sinusoidal. Determining the optimal drive current is difficult as it is linked with a variational problem. Many motor parameters and constraint conditions have to be considered in the analysis.

For simplifying the analysis in determining the optimal drive current, many literatures used the d-q model for analysis [1][2][3][4][5]. Such an analysis is based on the assumption that the waveform of the required current is sinusoidal. Using the d-q model, only the zero and first order harmonics of the armature winding inductances are considered. Using this model, the derivation of an optimal current is equivalent to the derivation of the amplitude and phase angle of the drive current for generating the required EM torque with minimum copper loss. In the following analysis, this sinusoidal current will be defined as Optimal Sinusoidal Drive (OSD) Current. OSD current has been widely used in many applications for reducing torque ripple and copper loss.

However, there is still a question: Is the real optimal drive current of the synchronous motor sinusoidal? If it is not, what is the true optimal current and how can it been determined? And what is the difference between the influences of the "real optimal drive current" and OSD current? The following analysis will present a method to get the real optimal drive current, and the answers of the questions can be found in the analysis.

II. ELECTROMAGNETIC TORQUE GENERATED IN THE OPERATION OF SYNCHRONOUS MOTOR

The EM energy stored in synchronous motor can be described as

$$E_{co}(\theta) = \frac{1}{2}[I]^T[L(\theta)][I], \tag{1}$$

where, [I] is the current in the motor windings, and [L(θ)] is the inductance of the windings. For the 3-phase synchronous motor,

$$[I]^T = [I_a, I_b, I_c, I_f], \tag{2}$$

and

$$[L(\theta)] = \begin{bmatrix} L_a(\theta) & M_{ab}(\theta) & M_{ac}(\theta) & M_{af}(\theta) \\ M_{ba}(\theta) & L_b(\theta) & M_{bc}(\theta) & M_{bf}(\theta) \\ M_{ca}(\theta) & M_{cb}(\theta) & L_c(\theta) & M_{cf}(\theta) \\ M_{fa}(\theta) & M_{fb}(\theta) & M_{fb}(\theta) & L_f(\theta) \end{bmatrix}. \tag{3}$$

In (2), I_f is the rotor field current which generates the exciting magnetic field through rotor field winding. In (3), $\mathbf{L_i}$ means the self-inductance of the i^{th} winding, and $\mathbf{M_{ij}}$ means the mutual-inductance between the i^{th} and j^{th} phase windings. The subscript **f** means the parameters related with the rotor winding.

The EM torque of the PMSM can be derived from its stored EM energy,

$$T_{em}(\theta) = \left.\frac{dE_{co}(\theta)}{d\theta}\right|_{[I]=Const} = \frac{1}{2}[I]^T \frac{d[L(\theta)]}{d\theta}[I]. \tag{4}$$

The $T_{em}(\theta)$ can be further expressed as,

$$T_{em}(\theta) = T_A(\theta) + T_R(\theta) + T_{Cg}(\theta). \tag{5}$$

where,

$$\begin{cases} T_A(\theta) = I_f \dfrac{d[M_f(\theta)]}{d\theta}[I_s] \\ T_R(\theta) = \dfrac{1}{2}[I_s]^T \dfrac{d[L_R(\theta)]}{d\theta}[I_s] + [I_s]^T \dfrac{d[M_S(\theta)]}{d\theta}[I_s] \\ T_{cg}(\theta) = \dfrac{1}{2} I_f^2 \dfrac{dL_f(\theta)}{d\theta} \end{cases}. \tag{6}$$

In above equations $\mathbf{T_A(\theta)}$ is the alignment torque of the synchronous motor which is linearly proportional to the stator current $\mathbf{[I_s]}$, and it is generated by the reaction between the stator field and rotor field. $\mathbf{T_R(\theta)}$ is the reluctance torque induced by the reaction between the stator current $\mathbf{[I_s]}$ and the field produced by the stator current. $\mathbf{T_{cg}}$ is cogging torque caused by the reaction between the rotor field produced by I_f and the rotor equivalent current.

In (6), the parameters are defined by

978-1-4244-4175-4/10/$25.00 ©2010 IEEE

$$\begin{cases}[M_f(\theta)] &= diag[M_{af}(\theta), M_{bf}(\theta), M_{cf}(\theta)] \\ [L_R(\theta)] &= diag[L_a(\theta), L_b(\theta), L_c(\theta)] \\ [M_S(\theta)] &= \begin{bmatrix} L_a(\theta) & M_{ab}(\theta) & M_{ac}(\theta) \\ M_{ab}(\theta) & L_b(\theta) & M_{bc}(\theta) \\ M_{ac}(\theta) & M_{bc}(\theta) & L_c(\theta)\end{bmatrix} \\ [I_s]^T &= [I_a(\theta), I_b(\theta), I_c(\theta)]\end{cases}. \quad (7)$$

For a required EM torque **T**, when the neutral line of the armature winding is not used, it seems that the optimal current for the 3-phase synchronous motor can be described with the following optimization equation with two constrained conditions

$$\begin{cases}\frac{1}{2}[I(\theta)]^T \frac{d[L(\theta)]}{d\theta}[I(\theta)] = T \\ I_a^2(\theta) + I_b^2(\theta) + I_c^2(\theta) = \min \\ I_a(\theta) + I_b(\theta) + I(\theta)_c = 0\end{cases}. \quad (8)$$

However, it is possible that, in some cases, no current can generate the required torque **T**. Therefore, it is more meaningful to use the following equation to replace (8) for describing the optimal drive current **[I(θ)]**,

$$\begin{cases}\{\frac{1}{2}[I(\theta)]^T \frac{d[L(\theta)]}{d\theta}[I(\theta)] - T\}^2 = \min \\ I_a^2(\theta) + I_b^2(\theta) + I_c^2(\theta) = \min \\ I_a(\theta) + I_b(\theta) + I(\theta)_c = 0\end{cases}. \quad (9)$$

From the constrained conditions to the drive currents in (8) and (9), we can know that,

$$I_c = -(I_a + I_b), \quad (10)$$

and one of the unknowns, I_c, in (9) can be eliminated when (10) is used. The current mode can thus be changed to,

$$\begin{cases}\{\frac{1}{2}[I(\theta)]^T \frac{d[L(\theta)]}{d\theta}[I(\theta)] - T\}^2 = \min \\ I_a^2(\theta) + I_b^2(\theta) + I_a(\theta)I_b(\theta) = \min\end{cases}. \quad (11)$$

The equation can be further rewritten as

$$\begin{cases}\{F + T + A_1 I_a + B_1 I_b - C_1(I_a + I_b) + A_2 I_a^2 + B_2 I_b^2 + C_2(I_a + I_b)^2 \\ \quad + M'_{ab} I_a I_b + M'_{bc} I_b (I_a + I_b) + M'_{ca}(I_a + I_b) I_a\}^2 = \min \\ I_a^2(\theta) + I_b^2(\theta) + I_a(\theta)I_b(\theta) = \min\end{cases}. \quad (12)$$

where,

$$\begin{cases}F = L'_f I_f^2 / 2 \\ A_1 = I_f \cdot dM_{af}/d\theta, B_1 = I_f \cdot dM_{bf}/d\theta, C_1 = I_f \cdot dM_{cf}/d\theta \\ A_2 = L'_a/2, B_2 = L'_b/2, C_2 = L'_c/2 \\ M'_{ab} = dM_{ab}/d\theta, M'_{bc} = dM_{ab}/d\theta, M'_{ca} = dM_{ca}/d\theta\end{cases} \quad (13)$$

and these coefficients are all the function of rotor position.

Using Lagrange multipliers method, the constrained optimal problem can be described as

$$\begin{aligned}&\{F + T + A_1 I_a + B_1 I_b - C_1(I_a + I_b) + A_2 I_a^2 + B_2 I_b^2 + C_2(I_a + I_b)^2 \\ &+ M'_{ab} I_a I_b + M'_{bc} I_b(I_a + I_b) + M'_{ca}(I_a + I_b) I_a\}^2 \\ &+ \lambda[I_a^2(\theta) + I_b^2(\theta) + I_a(\theta)I_b(\theta)] = \min\end{aligned} . \quad (14)$$

where, λ is a Lagrange multiplier to be solved.

There are three unknown values to be solved in (14), and they are I_a, I_b and λ.

To the both sides of (14), calculate the derivatives of I_a and I_b, separately, and rearranging the derived equations, the following simultaneous equations can be obtained,

$$\begin{cases}\lambda(2I_a + I_b) + [2(A_2 + C_2 - M'_{ca})I_a + \\ \quad (M'_{ab} - M'_{bc} + 2C_2 - M'_{ca})I_b][(A_2 + C_2 - M'_{ca})I_a^2 \\ \quad + (M'_{ab} - M'_{bc} + 2C_2 - M'_{ca})I_a I_b + (B_2 - M'_{bc} + C_2)I_b^2 - T] = 0 \\ \lambda(2I_b + I_a) + [2(B_2 + C_2 - M'_{bc})I_b + \\ \quad (M'_{ab} - M'_{bc} + 2C_2 - M'_{ca})I_a][(A_2 + C_2 - M'_{ca})I_a^2 \\ \quad + (M'_{ab} - M'_{bc} + 2C_2 - M'_{ca})I_a I_b + (B_2 - M'_{bc} + C_2)I_b^2 - T] = 0\end{cases}, \quad (15)$$

From (15), the Lagrange multiplier, λ, can be eliminated, and the following equation can be obtained which describes the relationship between I_a and I_b,

$$\begin{aligned}&2(M'_{ab} - B_2 + C_2 - M'_{ca})I_b^2 \\ &+ [2A_1 - B_1 - C_1 + 4(A_2 - B_2 + M'_{bc} - M'_{ca})I_a]I_b \\ &+ [A_1 - 2B_1 + C_1 + 2(A_2 - C_2 - M'_{ab} + M'_{bc})I_a]I_a = 0\end{aligned}, \quad (16)$$

If the airgap of the motor is smooth, both the reluctance and cogging torques of the synchronous motor are zero, like the case of the permanent magnet synchronous motor with surface mount magnet. In this case, the relationship between I_b and I_a determined by (16) can be simplified as

$$(2A_1 - B_1 - C_1)I_b + (A_1 - 2B_1 + C_1)I_a = 0, \quad (17)$$

and the solution to I_b is

$$I_b = \frac{A_1 - 2B_1 + C_1}{B_1 + C_1 - 2A_1} I_a, \quad (18)$$

If the back-emf of the motor is sinusoidal, i.e.,

$$\begin{cases}A_1 = I_f M'_{af}(\theta) = E_m Sin(p\theta) \\ B_1 = I_f M'_{bf}(\theta) = E_m Sin(p\theta - \frac{2\pi}{3}) \\ C_1 = I_f M'_{cf}(\theta) = E_m Sin(p\theta - \frac{4\pi}{3})\end{cases}, \quad (19)$$

the following result can be acquired,

$$I_b = \frac{Sin(p\theta - 2\pi/3)}{Sin(p\theta)} I_a, \quad (20)$$

where, p is the pole-pair of the motor,

When (18) and (19) are used to replace the related items in (14), it can be known that,

$$\begin{aligned}I_a &= \frac{-2T}{\sqrt{3}E_m[Cos(p\theta - \frac{2\pi}{3}) + Sin(p\theta - \frac{2\pi}{3})Cos(p\theta - \pi)Csc(p\theta)]} \\ &= \frac{2T}{3E_m} Sin(p\theta)\end{aligned}. \quad (21)$$

Therefore, for a 3-phase synchronous motor without reluctance torque and cogging torque, and the back-emf generated in the armature winding is sinusoidal, it can be known from (10), (20) and (21) that, the optimal drive current of the motor is,

$$\begin{cases}I_a = \frac{2T}{3E_m} Sin(p\theta) \\ I_b = \frac{2T}{3E_m} Sin(p\theta - \frac{2\pi}{3}) \\ I_c = \frac{2T}{3E_m} Sin(p\theta - \frac{4\pi}{3})\end{cases}. \quad (22)$$

that is , the drive current is also sinusoidal

The result is quite meaningful as it confirms that the sinusoidal current is reasonable to some synchronous motors. The optimal drive current used in these "idealized" motors is just the OSD current.

However, if the reluctance effects of the motor are considered, it means all the self and mutual inductances of the motor must be considered, the analysis becomes quite complicated. Will the optimal current still be sinusoidal for the synchronous motor?

III. Optimal Drive Current Of Reluctance Motor

For a reluctance motor, there is no winding on the rotor. Therefore, A_1, B_1, C_1 and F are thus zero; see (13). We can simplify (16) to

$$\begin{aligned}&(M'_{ab}-B_2+C_2-M'_{ca})I_b^2+2(A_2-B_2+M'_{bc}-M'_{ca})I_aI_b\\&+(A_2-C_2-M'_{ab}+M'_{bc})I_a^2=0\end{aligned} \quad (23)$$

The current I_b can be obtained by solving (23) directly, and the result is,

$$I_b=W\cdot I_a \quad (24)$$

where

$$\begin{aligned}W=&\frac{-1}{M'_{ab}-B_2+C_2-M'_{ca}}\{(A_2-B_2+M'_{bc}-M'_{ca})\pm[(A_2-B_2+M'_{bc}-M'_{ca})^2\\&-(A_2-M'_{ab}+M'_{bc}-C_2)(M'_{ab}-B_2+C_2-M'_{ca})]^{\frac{1}{2}}\},\end{aligned} \quad (25)$$

i.e., I_b is linear related with I_a, but the coefficient, W, varies with the variation of the rotor position. Using (24), the optimal current can be determined easily if I_a is known.

For (25), in the case where its denominator is zero, it can be proven that, the following two results can be obtained respectively,

$$\lim_{(M'_{ab}-B_2+C_2-M'_{ca})\to 0} W=\begin{cases}-0.5\\-1.5\end{cases} \quad (26)$$

In the reluctance motor, T_A and T_{cg} in (6) are zero, the EM torque contains only T_R, the reluctance torque component, which can be used to determine I_a with (6),

$$\begin{aligned}&A_2I_a^2+B_2I_b^2+C_2(I_a+I_b)^2+M'_{ab}I_aI_b\\&+M'_{bc}I_b(I_a+I_b)+M'_{ca}(I_a+I_b)I_a=T=T_R\end{aligned} \quad (27)$$

Using the relationship shown in (24), the following equation can be obtained,

$$(A_2+M'_{ab}W+B_2W^2)I_a^2-(M'_{ca}+M'_{bc})(1+W)I_a^2+C_2(1+W)I_a^2=T \quad (28)$$

And the solution of I_a is

$$I_a=\frac{\pm\sqrt{T}}{\sqrt{A_2-M'_{ca}(1+W)+C_2(1+W)^2+W[M'_{ab}+B_2W-M'_{bc}(1+W)]}} \quad (29)$$

Therefore, using (10), (24) and (29), the optimal drive current of a reluctance motor can be determined if the self and mutual inductances of the motor are known. The magnitude of the current is linearly proportional to the square root of the torque required.

Hence, if the parameters of the reluctance motor are known, motor optimal drive current for a reference torque can be calculated first and the result of the current can be saved in the memory of the driver. In the motor operation, the optimal current for other required torque can be easily determined by using (29).

In (24) and (29), two solutions are contained in the equations. It is not difficult to select the correct solution through the power loss comparison, current continuity, and that the value in the square root must be positive.

In [7], the coefficients shown in (13) are defined as the "identity" of the motor. In applying the optimal drive current to drive a reluctance motor, it is not required to get the curves of the inductance first and then calculate the "identity" of the motor. The identity can be obtained directly through test. The following testing steps can be used to obtain the identity of the reluctance motor,

i. Input the unit current to the A-phase winding of the motor. Keep the current be constant, and then rotate the rotor of the motor. During the rotation, measure the torque variation in one revolution. From (6), The torque curve obtained is $A_2(\theta)$ of (13); see (6) and (7);

ii. When $A_2(\theta)$ is known, $B_2(\theta)$ and $C_2(\theta)$ can be determined by using the following equations,

$$\begin{cases}B_2(\theta)=A_2(\theta-\frac{2\pi}{3p})\\C_2(\theta)=A_2(\theta-\frac{4\pi}{3p})\end{cases}; \quad (30)$$

iii. Input only the unit current to the A-phase and B-phase windings of the motor. With the current constant, rotate the rotor of the motor. During rotation, measure the torque variation in one revolution. The torque curve obtained is defined as $T_{ab}(\theta)$. From (6) and (7), M'_{ab} of (13) can be determined by the following equation,

$$M'_{ab}(\theta)=T_{ab}(\theta)-A_2(\theta)-B_2(\theta); \quad (31)$$

iv. When $M'_{ab}(\theta)$ is known, $M'_{bc}(\theta)$ and $M'_{ca}(\theta)$ can be determined by using the following equations,

$$\begin{cases}M'_{bc}(\theta)=M'_{ab}(\theta-\frac{2\pi}{3p})\\M'_{ca}(\theta)=M'_{ab}(\theta-\frac{4\pi}{3p})\end{cases}. \quad (32)$$

In the EM analysis, finite element (FE) method can also be used to calculate the torque $A_2(\theta)$ and $T_{ab}(\theta)$. Using the same steps introduced above, all the identity components can be obtained with FE method.

From (13), it can be known that, if the inductances of the motor contains a certain harmonics, the identity contains the same harmonics, though the amplitudes and phase angles of these two kinds of harmonics are different.

IV. Optimal Drive Current Of The Reluctance Motor With Fundamental Inductance

To investigate the effects of the optimal drive current obtained in the last section, its application to drive an "idealized" reluctance motor is analyzed. For this reluctance motor with p pole-pairs, its inductances contains only the fundamental harmonics, i.e.,

$$\begin{cases}A_2=2p\cdot L\cdot Sin(2p\theta)\\B_2=2p\cdot L\cdot Sin(2p\theta-2\pi/3)\\C_2=2p\cdot L\cdot Sin(2p\theta-4\pi/3)\\M'_{ab}=2p\cdot M\cdot Sin(2p\theta-\pi/3)\\M'_{bc}=2p\cdot M\cdot Sin(2p\theta-\pi)\\M'_{ca}=2p\cdot M\cdot Sin(2p\theta-5\pi/3)\end{cases}. \quad (33)$$

Replace the related items in (29) with the ones of (33), the following result can be obtained,

$$I_a=\pm\frac{2\sqrt{T}}{\sqrt{3(L+M)}}\cdot Sin(p\theta+\pi/4), \quad (34)$$

From (34), it can be found that, for such a reluctance motor, the phase angle of its OSD current is not related to the motor parameters and load, and is a constant value. However, the amplitude of the OSD current is linked with the motor inductances, and load torque.

From (10) and (24), the following result can be further obtained,

$$\begin{cases} I_a = \pm \dfrac{2\sqrt{T}}{\sqrt{3(L+M)}} Sin(p\theta + \pi/4) \\ I_b = \pm \dfrac{2\sqrt{T}}{\sqrt{3(L+M)}} Sin(p\theta - \dfrac{5\pi}{12}) \\ I_c = \pm \dfrac{2\sqrt{T}}{\sqrt{3(L+M)}} Sin(p\theta - \dfrac{13\pi}{12}) \end{cases}, \quad (35)$$

Therefore, for the "idealized" reluctance motor, its optimal drive current is sinusoidal. This current is also an OSD current. It can thus be understood that using sinusoidal current to reduce the torque ripple and motor vibration is an interesting topic [8].

V. Optimal Drive Current Of A Switched Reluctance Motor

Switched reluctance motor is a motor whose inductance varies with the changing of the rotor position, and normally driven with pulse model. This pulse model is easy to realize, but can induce rich torque ripple in motor operation. For verifying the effectiveness of the optimal drive model obtained in above sections, here, the motor shown in Fig. 1 is used in the analysis. This is a 3-phase motor. Through the performance analysis, the effective reluctance pole-pair of the motor is 8. The Star-connection is used for the 3-phase motor armature windings, and there is no neutral line.

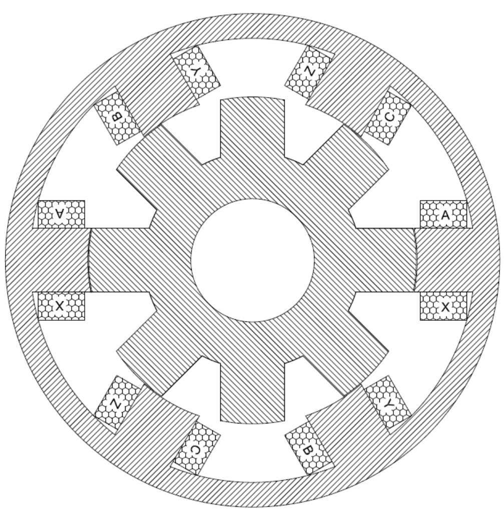

Fig. 1 SRM1: A switched reluctance motor

FE method is used to calculate the "identity" of the motor. The motor will be called as SRM1 in short. Fig. 2 shows the magnetic field distribution of the motor in at 0°, where, only A-phase winding is excited. From the field distribution at different rotor positions with the constant I_a, $A_2(\theta)$ can thus obtained with FEM. In the same way, when A-phase and B-phase were excited with the same constant current, $T_{ab}(\theta)$ can be calculated. Using the calculation steps introduced in Section III, all the components of SRM1 "identity" can thus be obtained, and Fig. 3 shows the curves of the $A_2(\theta)$ and $M'_{ab}(\theta)$. Through Fourier series analysis, all the harmonics of the A_2 and M'_{ab} can be known, and the major items are shown in TABLE I.

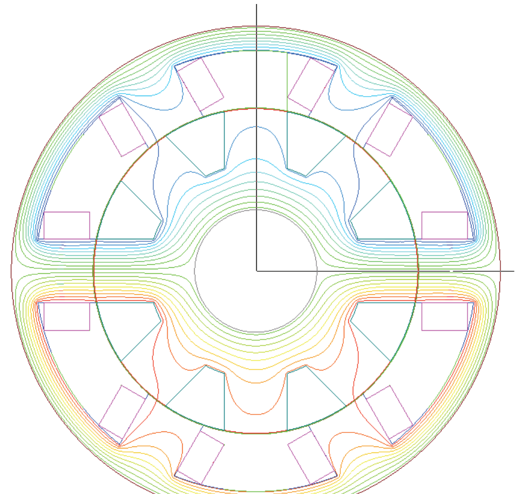
Fig. 2 The magnetic field distribution in SRM1when the rotor is at 0 degree and only A phase is excited

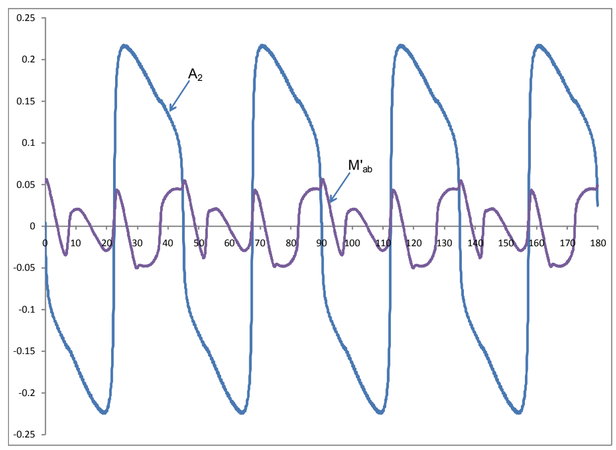

Fig. 3 The $A_2(\theta)$ and $M'_{ab}(\theta)$ of SRM1

TABLE I The Harmonics Amplitude of SRM1 Identity

Identity (mNm/A^2)	1st Order	2nd Order	3rd Order	4th Order	5th Order	6th Order
A_2	219.85	46.634	66.59	18.89	34.82	12.07
M'_{ab}	21.384	23.496	26.46	11.54	4.22	12.83

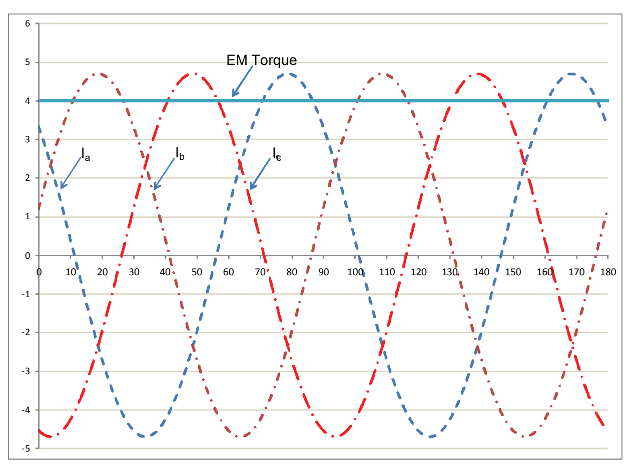

Fig. 4 Case-1: The optimal drive current and the EM torque generated by the current (Only the fundamental identity is considered)

In order investigate the effects of the optimal drive current, four motor cases will be analyzed. In Case-1, only the fundamental harmonics of the identity are considered, i.e., SRM1 is considered as an idealized reluctance motor. As it was mentioned in Section IV, in this case, the optimal drive current is sinusoidal, and its expression is shown in(35) . Fig. 4 shows the EM torque generated by the current, where, **T** in (29) is set at 4 Nm. To simplify the curves, the results in the position range 0° to 180° are shown in the figure. It is same as the conclusion obtained in Section IV, using the OSD current and the optimal drive current, no torque ripple is induced in the motor operation.

In the Case-2, only the 1^{st} and 2^{nd} order harmonics of the identity are considered, i.e., the motor inductances are considered formed only by the 1^{st} and 2^{nd} order harmonics. In the calculation, the torque T is still set at 4 Nm. The torque generated by the optimal drive current and OSD current are shown in Fig. 5. The 3 phase optimal drive currents are also shown in the figure. In this case, as it is shown in Fig. 5, OSD current cannot generate constant torque, but the optimal drive current can still eliminate the torque ripple. The effects of the latter can thus be fully revealed and confirmed.

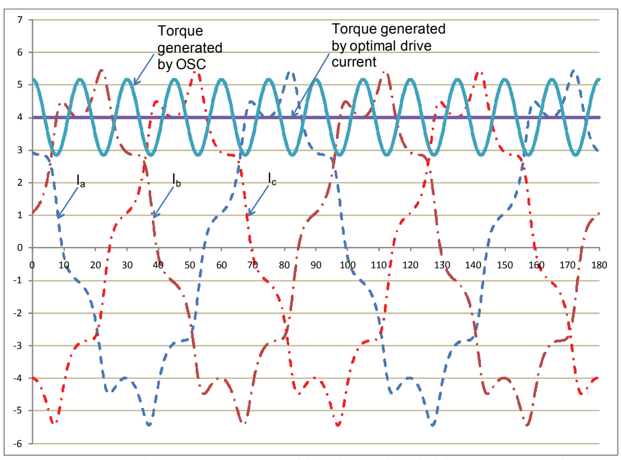

Fig. 5 Case-2: The torque generated by the optimal drive current and OSD current (Only the 1^{st} and 2^{nd} order identity harmonics are considered)

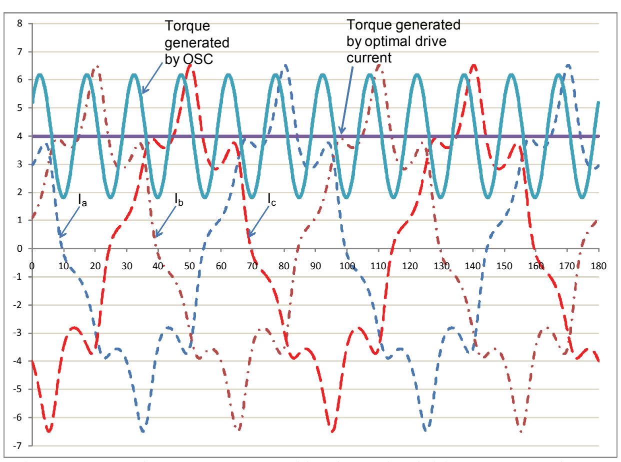

Fig. 6 Case-3: The torque generated by the optimal drive current and OSD current (Only the 1^{st} to 3^{rd} identity harmonics are considered)

For the 3-phase motor, the triple order torque ripple is normally considered as the ones which are difficult to be eliminated with drive currents. What is the effect of the optimal drive current to this kind of torque ripple? In the Case-3, 3^{rd} harmonic is considered. That is, in this case, SRM1's inductances are considered to be formed only by the 1^{st}, 2^{nd} and 3^{rd} harmonics. Simulation results show that the optimal drive current can still generate constant EM torque when the 3^{rd} order harmonics are considered. The current waveform and the torque generated are also shown in Fig. 6. From the figure, it can be seen that the OSD current can't reduce the torque ripple. Comparing with the result of Case-2, the existence of the 3^{rd} order harmonics identity worsens the torque ripple introduced by the OSD current.

What will happen when all the harmonics of the identity are considered? i.e., what is the result to the "real" SRM1? This is Case-4, where, the identity shown in Fig. 3 is used directly in the calculation. The optimal drive current, and the torques generated by OSD and optimal drive currents, are shown in Fig. 7. Comparing the results shown in Fig. 7 and the ones obtained in the Case-1 to Case-3, it can be known that the rich harmonics of the motor identity make the torque ripple be more complicated when the OSD current is used. However, the effectiveness of the optimal drive current is confirmed again in the Case-4.

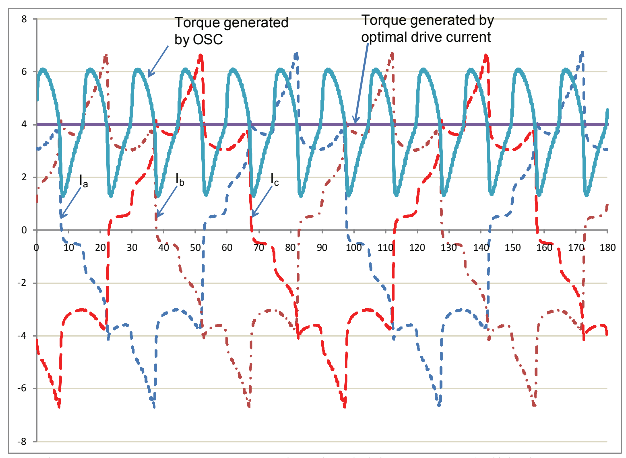

Fig. 7 Case-4: OSD current and optimal drive current (All inductance harmonics are considered)

For making the influence of high order inductance harmonics be clear, TABLE II shows the ratios of torque ripple of the 4 cases separately. The torque ripple ratio, R_{tr} is defined as

$$R_{tr} = \frac{T_{max} - T_{min}}{2T_{ave}}, \qquad (36)$$

where, T_{max} and T_{min} are the maximum and minimum torque during the motor operation. T_{ave} is the average torque in one revolution.

TABLE II The Torque Ripple Ratio of SRM1 In The Four Cases

Case	1	2	3	4
R_{tr} of the OSD current	0.00%	28.80%	53.23%	58.95%
R_{tr} of the optimal drive current	0.00%	0.00%	0.00%	0.00%

TABLE III shows the copper loss of the optimal drive currents in the 4 cases, where, the loss of the OSD current is used as the reference for the comparing. As the OSD current in the four cases are same, it is no need to compare its losses in these four cases.

TABLE III The Copper Loss of SRM1 In The Four Cases

Case	1	2	3	4
Copper loss induced by optimal drive current	1.0000	1.0219	1.1285	1.0825

From TABLE II, it can be found that, as more harmonics are included in the motor inductances, the torque ripple is induced by OSD current worsened correspondingly.

Comparing the optimal drive current waveforms shown in Fig. 4 to Fig. 7, it can be known that, when the harmonics of the identity become richer and richer, the waveforms of the current become more and more complicated. TABLE III shows that, generally, the complexity of the current increases the copper loss. However, from the constrained conditions shown in (9), it can be known that the optimal drive current can keep the EM torque constant with minimum copper loss. Here, the loss of OSD current is just used as a reference. Comparing directly the losses of the optimal drive current with OSD current bear little significance as the latter introduces torque ripples as in Case-2 to Case-4.

VI. CONCLUSIONS

The simulation results have confirmed the effectiveness of the general model presented for calculating the optimal drive current of synchronous motor. Using the model, it can be proved that, when the reluctance torque is zero and back-emf varies sinusoidal in space domain, the optimal drive current is sinusoidal. It can also be proved that, for the reluctance motor containing only the fundamental inductance component, its optimal drive current is still sinusoidal. Both of these results are consistent with the traditional considerations in the electric machine analysis.

The "identity" of the motor is a very useful concept in determining the optimal drive current, and is easy to be obtained through FE analysis and testing. Analysis shows that, when the identity of the reluctance motor contains high order harmonics, the optimal drive current of the motor is not sinusoidal, and the sinusoidal current is very weak in reducing the torque ripple. However, using the presented current model, the torque ripple can still be eliminated with the lowest copper loss. The effectiveness of the presented model and analysis method has been verified in many applications.

REFERENCES

[1] S. Shinnaka, T. Sagawa, "New Optimal Current Control Methods for Energy-Efficient and Wide Speed-Range Operation of Hybrid-Field Synchronous Motor", IEEE Trans. on Industrial Electronics, Volume 54, Issue 5, Oct. 2007, pp. 2443–2450.

[2] P.L. Chapman, S.D. Sudhoff, C.A. Whitcomb, "Optimal current control strategies for surface-mounted PM synchronous machine drives", IEEE Trans. on Energy conversion, Volume 14, Issue 4, Dec 1999, pp.1043–1050.

[3] Malekian K., Sharif M.R., Milimonfared J., "An optimal current vector control for synchronous reluctance motors incorporating field weakening", 10th IEEE International Workshop on Advanced Motion Control, Volume , Issue , 26-28 March 2008, pp.393–398.

[4] Bianchi N.; Bolognani S., Zigliotto M., "Time optimal current control for PMSM drives", IECON 02, 1 Nov. 2002.

[5] Chapman P.L., Sudhoff S.D., Whitcomb C.A, "Torque ripple minimization in permanent magnet synchronous servo drive", IEEE Trans. on Energy conversion, Volume 14, Issue 3, Sep 1999, pp.616–621.

[6] C. Bi, Q. Jiang, S. Lin, T.S. Low, A.A. Mamun, "Reduction of acoustic noise in FDB spindle motors by using drive technology", IEEE Trans. On Magnetics, Vol. 39, March 2003, pp. 800-805.

[7] T.S. Low, Bi Chao, K.T. Chang, "'Motor identity'-a motor model for torque analysis and control", IEEE Trans. on Industrial Electronics, Vol., 43, No. 2, April 1996, pp. 285-291;

[8] X. Ojeda., X. Mininger, M. Gabsi, M. Lecrivain, Sinusoidal Feeding for Switched Reluctance Machine: Application to Vibration Damping. International Conference on Electrical Machine, ICEM 2008, Vilamoura, Portugal 6-9 September 2008

VI. BIOGRAPHIES

Chao BI got B.Eng from Hefei University of Technology, M.Eng from Xian Jiaotong University, and Ph.D. from National University of Singapore. He is working at Data Storage Institute (DSI), A*Star as senior scientist. His special fields of interest include electric machine design and drive, electromagnetic field analysis, and optimization technology. . He has published more than 120 papers, and co-authors of one book in these areas.

Hla Nu PHYU received the B.Eng from Yangon Technological University, Myanmar in 1998 and Ph.D. from National University of Singapore in 2005. She is currently working as a Senior Research Fellow for Data Storage Institute, and research interests include computational electromagnetics, numerical techniques, design and analysis of PM motors.

Cheng Su SOH received the B.Eng with First Class Honours, M.Eng and Ph.D from the National University of Singapore. He is currently working in Data Storage Institute working in the area of motor design and control, embedded and hardware system design.

Quan JIANG received his B.Eng. degree from Hefei University of Technology, M. Eng. and Ph.D. degrees from Southeast University, China. He is working as a Research Scientist in DSI. His research interests are in design, control and testing of electric machines, EM field analysis, switched reluctance motor and drives. He has published more than 80 academic papers, and co-authors of two chapters of two books.