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Optimal Spline Data Fitter and its Application in the Dynamic Speed Measurement of BLDC Motor

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Abstract: An effective data fitter is presented which can restore the signal from the measurement results corrupted by noisy. Its application in the sensorless speed measurement of BLDC motors is also introduced.

Key words: Motor Speed measurement, Hard-disk Drive, Optimal Spline, Data Smoothing

1. Introduction

Measuring accurately the transient rotation speed is important in the research and applications of electrical machine. Generally speaking, there are two common ways of obtaining it, by means of position sensor, e.g., encoder and resolver, or sensorless measurement. In both of these methods, the speed signal obtained is a discrete sequence. Besides, error is inevitable in either method and the speed obtained might undulate with respect to time. Additionally, there are also many researches and applications which require both the continuous speed signal and the acceleration value. As such, the discretized speed sequence can not be used directly. Some signal processing methods must be used to process the speed signal data obtained.

Discrete Fourier Transform (DFT[1]) or its faster counterpart Fast Fourier Transform (FFT) is widely used in the signal processing, but they are not suitable in processing the transient speed signals as the signals vary fast and not cyclic. Moreover, the input data number of FFT must be 2^n, which delimits the application range of it, and the filtered signal by either DFT or FFT will probably contain serious error around the side bound due to the aperiodic nature of the speed signal.

Another common method of data processing is spline interpolation [2]-[4]. Nevertheless, if the data entry is massive, and data values contain errors, the spline algorithm is very computational expensive on the ground that the every little segments between the two consequent data sites has to be evaluated, and the resultant interpolant might still fluctuate.

This paper introduces a new algorithm, which, by effectively making use of the entire data sites, removes noise and gives the smooth 1st order derivative of the original discrete signal while preserving the macroscopical behavior, as well as its implementation in the application of dynamic speed measurement of the sensorless brushless DC (BLDC) motor in hard disk drive (HDD).

2. Proposed approach

2.1 Algorithm development

As is well known, spline interpolation is a widely used tool in building up continuous analytical expression from the discrete data samples.

In our application, the objective is to rebuild the global trends of speed and acceleration. This implies that the curve rebuilt should be C^2 continuous because the speed and acceleration in the real world can never change abruptly with the advent of inertia. As such, cubic spline interpolation fits the C^2 continuous requirement. But, in real practice, there could be huge data points captured. It is unwise and unnecessary to have so many segmented functions if only the global trend is of our interest.

Moreover, so far as spline interpolation is concerned, in order to carry out the complete or clamped cubic spline interpolation, which might lead to a more accurate result, the boundary conditions, e.g., the 1st derivatives at the edge points must be given in those processes. However, these boundary conditions are not always known. As such, for simplicity, sometimes nature spline interpolation which assumes that those 1st derivative values equal zero is also used instead of the clamped one. But, in some application, e.g. the transient speed measurement of the motor, this kind of assumption might not be in reason because the time instant we begin data acquisition might not be the very time instant motor begins coasting, which implies the initial acceleration in the sampled speed is not zero. Furthermore, since the motor is coasting in a degressive deceleration manner with the advent of circumstance viscosity, the nature cubic spline assumption might introduce other kinds of noise into the resultant signal reconstructed. Hence, natural spline should not be used. Then, how can determine the optimized value based on the data extent together with the 1st derivative at the side edges to minimize the global error compared to the original data sites?

Here comes the algorithm, which builds a bridge linking the original raw noisy discretized speed data with its continuous counterparts and its 1st derivatives as well. In this algorithm, cubic spline interpolation, which just meets the C^2 continuous constraints, is used indirectly.

Based on previous discussion, the basic idea in the new algorithm is that with massive data entry, it is unnecessary to use all these points as the interpolation breaks. Further, because the data points are corrupted seriously with noise, it is also unwise to use the interpolation directly. With these considerations, the whole data entry is evenly divided into some segments. Yet, these segment points need not to be just at the data sites as it is shown in the following figure.

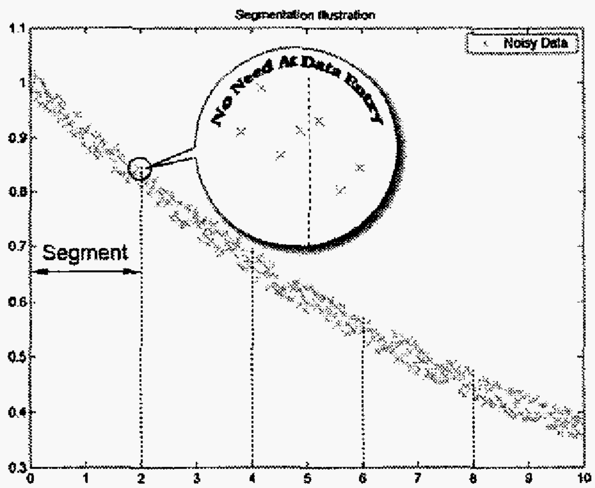

Fig. 1 Segmentation Illustration

Nevertheless, because the requirement of evaluating the 1st derivative values of the data sites in our research of dynamic speed, the clamped spline is thereafter utilized instead of nature one. With these segments and boundary condition given by the 1st derivative at the edge data points, the interpolation can be carried out along the whole data extent.

Given the segment number N, we will have N+1 edges on these segments, thereafter defined as knot and denoted as x_k. Here, the subscript of the knots starts from 0 and ends at N, while that of the segments stars from 1 and ends as N. Usually, the cubic spline interpolant in the interval $[x_k, x_{k+1}]$ can be express as follows provided with the 1st derivative value, denoted as m_k on these knots, where v_k is the function value, i.e. 0th derivative, at the knot, h_k is related with the segments' length and x is the point to be interpolated.

$$S_k(x)=m_{k-1}\frac{(x_k-x)^2(x-x_{k-1})}{h_k^2}-m_k\frac{(x-x_{k-1})^2(x_k-x)}{h_k^2}+v_{k-1}\frac{(x_k-x)^2\left[2(x-x_{k-1})+h_k\right]}{h_k^3}+v_k\frac{(x-x_{k-1})^2\left[2(x_k-x)+h_k\right]}{h_k^3} \qquad (1)$$

With the C^2 continuous constraints, m_k can be solved via the following equations if the boundary conditions, i.e. 1st derivative values on the very left and right edges of the entire data sites are given as in the clamped spline interpolation. As long as m_k is obtained, the interpolant is uniquely determined and we can calculate any point value within the data range according to the polynomial function (1).

$$\begin{bmatrix} 2 & \mu_1 & 0 & \cdots & 0 & 0 & 0 \\ \lambda_2 & 2 & \mu_2 & \cdots & 0 & 0 & 0 \\ 0 & \lambda_3 & 2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2 & \mu_{N-3} & 0 \\ 0 & 0 & 0 & \cdots & \lambda_{N-2} & 2 & \mu_{N-2} \\ 0 & 0 & 0 & \cdots & 0 & \lambda_{N-1} & 2 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \\ m_{N-3} \\ m_{N-2} \\ m_{N-1} \end{bmatrix} = \begin{bmatrix} c_1-\lambda_1 m_0 \\ c_2 \\ c_3 \\ \vdots \\ c_{N-3} \\ c_{N-2} \\ c_{N-1}-\mu_{N-1} m_N \end{bmatrix} \qquad (2)$$

The μ_k and λ_k is determined by the segment length of the whole interval. If the segment is divided evenly, $\mu_k = \lambda_k = 0.5$. And c_k is calculated from the function value on the knots as is in the following form.

$$c_k = 3\lambda_k\frac{v_k - v_{k-1}}{h_k} + 3\mu_k\frac{v_{k+1} - v_k}{h_{k+1}} \qquad (3)$$

We now have the unknown variables $v_0 \sim v_N$, m_0 and m_N. Thus, the interpolation can not be carried out. The next step is the optimization problem to find out these values with which we can minimize the following L_2 Norm, where p is the iteration index within that segment and y is the original data corrupted with noise.

$$E(v_0, v_1, \ldots, v_N, m_0, m_N) = \sum_{k=1}^{N} \sum_{p} [S_k(x_p) - y_p]^2 \quad (4)$$

The above equation simply expresses the deviation between the resultant cubic spline interpolant, determined by the variables $v_0 \sim v_N$, m_0 and m_N, with the original data points. If above energy can be minimized, we can thus obtain the optimized cubic spline interpolant from the variables, $v_0 \sim v_N$, m_0 and m_N in (1). By the energy minimization, the partial differential of the L_2 Norm with respect to these variables $v_0 \sim v_N$, m_0 and m_N should be zero as is shown in the following equations.

$$\frac{\partial E}{\partial v_i} = 2\sum_{k=1}^{N} \sum_{p} [S_k(x_p) - y_p] \frac{\partial S_k(x_p)}{\partial v_i} = 0 \quad (5)$$

$$\left.\frac{\partial E}{\partial m_j}\right|_{j=0,N} = 2\sum_{k=1}^{N} \sum_{p} [S_k(x_p) - y_p] \frac{\partial S_k(x_p)}{\partial m_j} = 0 \quad (6)$$

From the matrix in (2), we can know that the 1st derivative value $m_1 \sim m_{N-1}$ has linear relationship with, i.e. can be linearly represented by, the objective function value on the knots, $v_0 \sim v_N$, and boundary condition, m_0 and m_N. Therefore, m_k can be rewritten in the following.

$$m_k = \sum_{i=0}^{N} a_{ki} v_i + b_k m_0 + c_k m_N \;, k = 1, \cdots, N-1 \quad (7)$$

Bring these m_k into (2) and differentiate both sided with respect to $v_0 \sim v_N$, m_0 and m_N. These coefficients, $\{a_{ki}, b_k, c_k\}$ can be computed easily instead of solving the inverse of the left square matrix and matrix multiplication.

Given linear representation coefficients, the spline interpolant in equation (1) can also be expressed as the linear combination of $v_0 \sim v_N$, m_0 and m_N. Hence, S_k can be written in the following form.

$$S_k = \sum_{i=0}^{N} A_{ki}(x, x_k, x_{k-1}) v_i + B_k(x, x_k, x_{k-1}) m_0 + C_k(x, x_k, x_{k-1}) m_N \quad (8)$$

From equation (5) and (6), iterating i from 0 to N, altogether, we can get N+3 simultaneous equations corresponding to the objective variables, $v_0 \sim v_N$, m_0 and m_N. Hence, bringing in every {x, y} value set, i.e. the raw data site, into (5) and (6), and solving this N+3 linear system, the optimized value of $v_0 \sim v_N$, m_0 and m_N can be obtained.

The final step of optimal spline process is here to use these optimized function value on each knot together with optimized boundary condition to do spline interpolation along the whole data scope, which concatenates these artificially divided segments. Thereafter, any point within the whole data range can be evaluated by the resultant interpolant. Since the cubic piece-wise polynomial is utilized to represent the original data, the 1st derivative of any data point within the scope can be calculated as well according to the reconstructed interpolant. Besides, the function and 1st derivative curve is smooth.

2.2 Simulation Results

With the foregoing algorithm, some routines have been written to verify the results of this approach.

First of all, we use the sinusoid waveform corrupted with random noise as the input to the optimal spline data fitter, the algorithm above. In detail, the data range starts from 0 to 10 with the interval length of 0.01, and is divided into 10 segments altogether, as is shown in Fig. 2 (N = 10), which means there are 11 knots and 13 variables to be optimized with 1001 data entries as input according to above algorithm. The frequency of the sine wave is $\pi/5$, which implies the curve within the data range contains only 1 period. Besides, the noise level is 5% of the amplitude of the sine wave, i.e. 1.

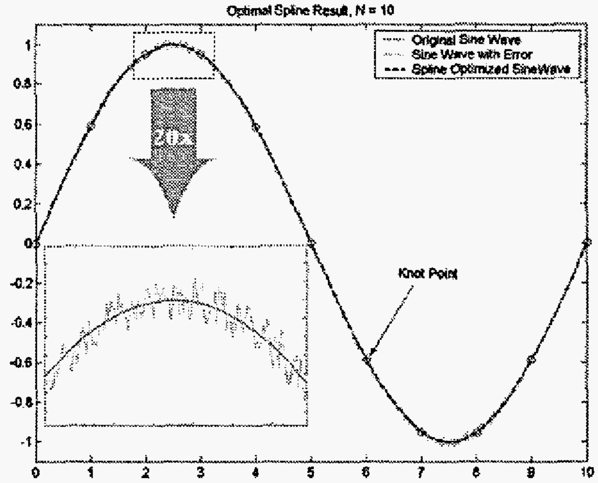

Fig. 2 Noisy Sine Wave Fitted Result

From the above figure, it is clear that the global behavior of the noisy sine has been retained while the noise wiped out. The resultant interpolant stays right in the center of the original fluctuating signal. The accuracy of the fitter to the signal value is quite satisfied.

The resultant 1st derivative of the optimized noisy sine wave is also shown as follows accompanied with the actual 1st derivative curve for analytical equations. From the figure, the fitter can also give satisfied result to the 1st derivatives of this sinusoidal signal.

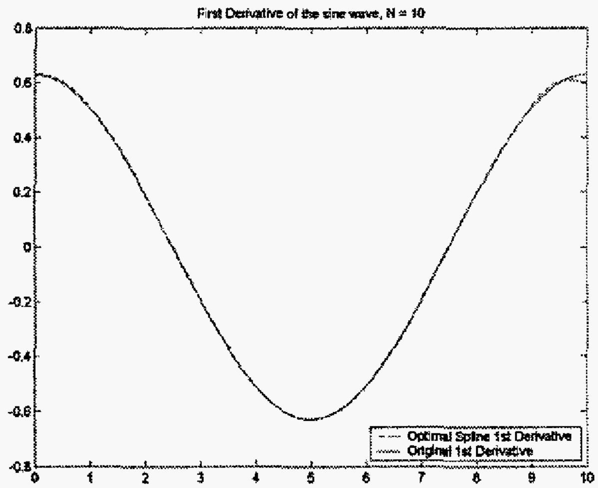

Fig. 3 Optimal Spline 1st Derivative Curve (Sine)

The exponential is another basic function widely used in scientific research and application, which will also be evaluated in our case for the analysis of this algorithm. The exponential surveyed is in the form of *f(x) = exp(-0.1x)* while the noise level is 0.02, i.e. the maximum stochastic noise amplitude is 0.02. The input data entry starts from 0 to 10 with the interval length of 0.0005, i.e. altogether there are 20001 points as input. Besides, the segment number for optimal spline calculation is selected to be 10 as is same with the case in sine simulation. And the 1st derivative is also analyzed by comparing with the original analytical curve without noise. The results are shown in Fig. 4 and Fig. 5 respectively.

Clearly, again, the resultant data fitter retrieves the global behavior from the original noisy data and reconstructed the interpolant right in the middle of the noisy signal, which is due to the use of least-square optimization in the optimal spline algorithm. Concerning the 1st derivative of the noisy exponential data, the interpolant can also follow the original derivative curve.

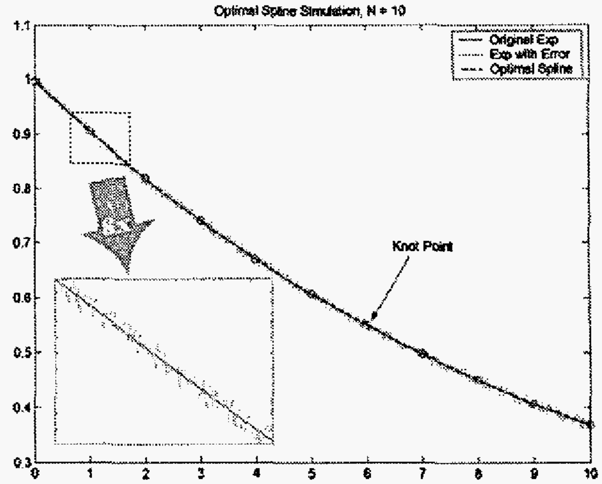

Fig. 4 Noisy Exponential Wave Fitted results

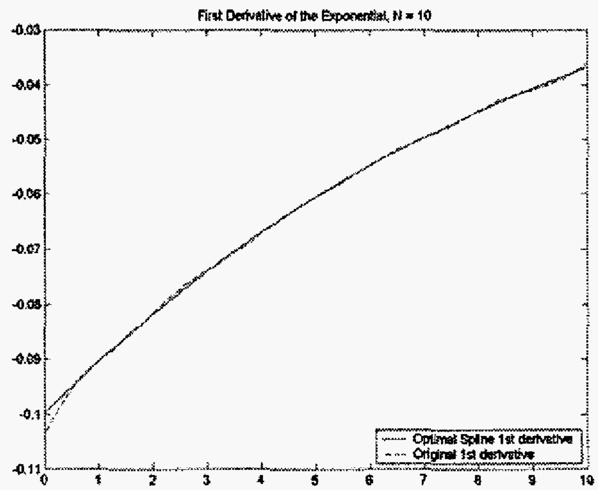

Fig. 5 Optimal Spline 1st Derivative Curve (Exp)

3. Practical Implementation

With the above simulations, the data fitter proposed behaves well in tracking the original signal global behavior despite the noise imposed on it. It is time for real practice.

3.1 Speed signal obtained in BLDC motor

In HDDs, the spindle motor used to drive the disk is nothing but a BLDC motor. Sensorless BLDC control has long before become the mainstream due to the compactness characteristic of the HDD.

Knowing and evaluating the effects of the dynamic speed is very important in analyzing the performance of the HDDs. When the sensorless BLDC mode is used to drive the spindle motor, the speed signal can be obtained roughly by detecting the motor's back electro-motive force (Back-EMF) induced in the motor winding. We can know the rotor positions where the Back-EMF signals cross zero. For the 3-phase motors, $6P$ rotor positions in one revolution can be known in the measurement, where, P is the pole-pair of the motor. The rotor speed can thus be known by computing the difference of the nearby rotor positions. In Fig. 6, which is locally amplified, illustrates the raw calculated speed signal in a freewheeling test.

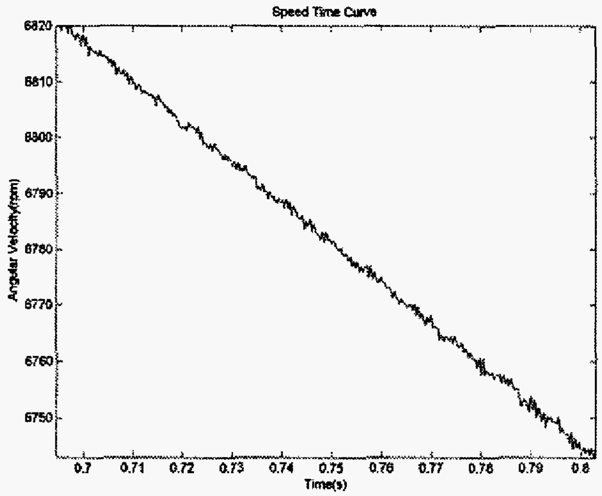

Fig. 6 Raw Speed Illustration

From the figure, the speed obtained in this procedure is corrupted by errors, not just by the error of the difference, but also the errors caused by the distortion of Back-EMF waveform, effects of armature reaction, and rounding or quantization of sampling card even the noise in the data transmission. The rotational speed acquired in this manner can only shows us the values in discrete rotor positions and might not be suitable for further analysis of the motor speed. All these caused serious problems in our research on the transient analysis of BLDC motor.

3.2 Digital Fitter and Optimal Spline

As is shown in Fig. 6, the initial speed signal of the BLDC motor is dithering because of the ubiquity of errors. However, from the total speed data obtained, we can still feel the entire behavior of the speed signal which is accelerating, or decelerating, with time passing. The task now is to filter out the noise, preserve the signal and continuously express the signal in analytical forms for further analysis.

Optimal spline process, the aforementioned algorithm rewritten in C++ due to the implementation in our research, is carried out to build up the data fitter to process the speed signal and give the continuous representation of the speed as well. Fig. 7, which is also magnified locally, gives a vivid expression of the data fitter results.

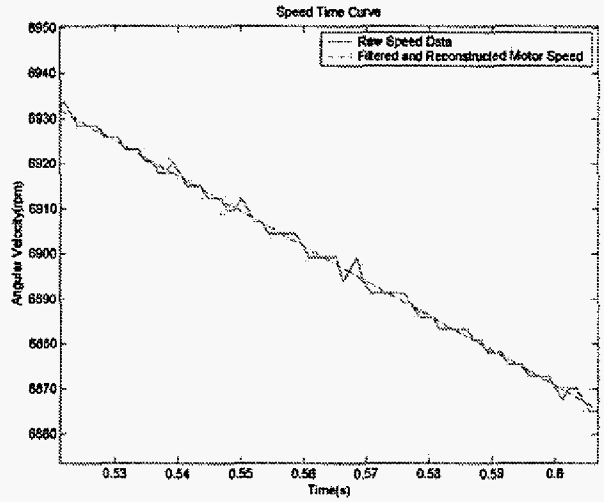

Fig. 7 Optimal Spline Fitted Speed

3.3 Acceleration

In many cases, acceleration, i.e. 1st order derivative of the speed signal, is required in analysis. Nevertheless, with the initial speed data obtained as is shown in Fig. 6, the acceleration can not be calculated correctly due to the errors as well as undulance mentioned in above sections. But, the developed data fitter based on the *Optimal Spline fitter*, $S(x)$, makes the acceleration signal curve obtainable from the utilization of the original speed signal.

As is stated above, $S(x)$ is C^2 continuous in the test range. The 1st order derivative of $S(x)$, i.e., the acceleration of the motor speed, is not only continuous but also smooth in the data range. This means that we can get smoothly the transient acceleration of the motor by using the data fitter developed. In our research, the resultant acceleration speed, which is calculated directly by differentiating the interpolant, i.e. piece-wise polynomial functions, is depicted as follows. Apparently in the figure, the curve of the acceleration is smooth enough to represent the reality situation in our experiment.

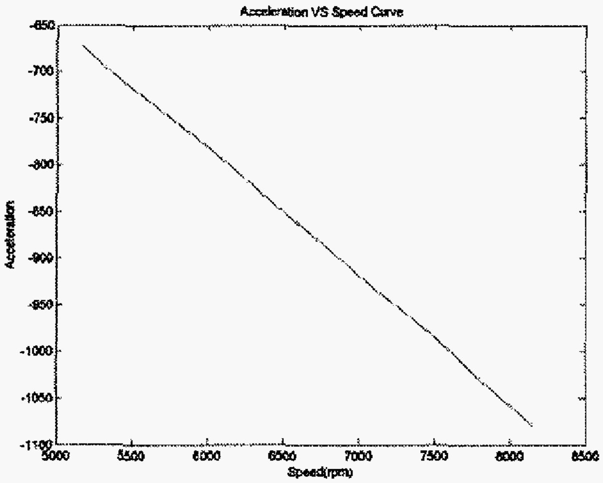

Fig. 8 Acceleration Deducted by Optimal Spline (Speed Domain)

For making the result be clear, only the results in the speed range around 5,000 to 8,000 rpm are displayed (speed domain in the above figure). Fig. 9 shows the same acceleration results in the time domain. The negative acceleration results in Fig. 8 and Fig. 9 explain that, in the freewheeling test, the motor speed is decelerated degressively.

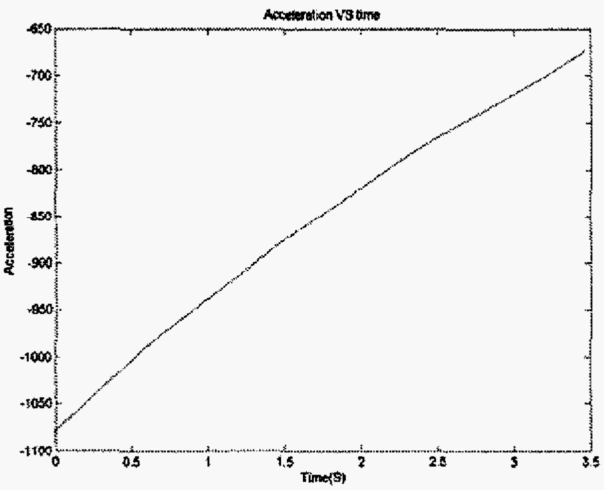

Fig. 9 Acceleration Deducted by Optimal Spline (Time Domain)

The C^2 continuous constraints brought by the *Optimal Spline* makes the reconstructed acceleration curve even and much closer to the acceleration behavior in the reality, which is a must for some transient and dynamic speed analysis in our research.

4. Conclusions

A new algorithm for the reconstruction of smooth curve as well as smooth 1^{st} derivative and data fitting from noisy data, together with its application is introduced and analyzed in this paper. Moreover, the simulation results and experimental results of the data fitter are also demonstrated. Both the simulation and experimental results show that the data fitter built up is robust in removing the noise from the signal and can also generate satisfactory continuous and smooth signal its self as well as the 1^{st} derivative of the signal from the original noisy signal. It is a very useful tool used in our research in analyzing the performance of sensorless BLDC motors.

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