Estimation of Back-EMF of PM BLDC Motors Using Derivative of FE Solutions

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***Abstract*—Based on finite element analysis, an effective method is developed to estimate the back-emf produced in electric machines during operation. For a given rotor position, the method utilizes only one finite element solution in its computation, and no numerical derivative is required. The accuracy of the back-emf estimated can thus be improved, and the computing time can be reduced. Computational results obtained from the proposed method to calculate the back-emf of permanent magnet brushless DC motor is introduced in the paper.**

***Index Terms*—Finite element, back-emf, motor.**

I. INTRODUCTION

FINITE Element (FE) method is a powerful tool in the analysis of the electromagnetic (EM) field in electric machines. Based on the FE analysis, the distribution of the magnetic potential can be found, and EM characteristics of the machine can thus be derived through the post-processing of FE analysis. However, many of such post-processing procedures are time consuming, and the accuracy of the results varies from case to case depending on the type of numerical methods applied. In the paper, the discussion will be around the estimation of the voltage-constant of the machine, and the back-emf of electric machines generated in the machine operation by using the results of FE analysis.

It is well known that $\boldsymbol{E}$, the back-emf generated in the armature winding is caused by the variation of the flux linked with the winding, and their relationship can be expressed as,

$$E = -\frac{d\psi}{dt}. \tag{1}$$

where, ψ is the flux-linkage of the armature winding.

For simplifying the explanation, the following analysis focuses on the no-load operation of permanent magnet (PM) brushless (BLDC) motors. In such cases, the armature current of the motor is zero, and the EM field of the machine is only produced by the magnet poles mounted on the rotor.

Using FE mesh, the flux linkage of the winding can be expressed as

$$\psi = \sum_e \psi_e = \sum_e w_e \phi_e. \tag{2}$$

where, ψ_e is the flux-linkage associated with the winding in the element e, ϕ_e and w_e are the flux and winding turns in e, respectively.

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For a 2D EM problem, it is well known that the flux can be expressed by using the distribution of the vector magnetic potential $\boldsymbol{A}$ in the problem region, that is

$$\varphi_e = A_{ei} - A_{ej}. \tag{3}$$

where, the subscripts i and j represent the position pair located at the two ends of the coil. A_{ei} and A_{ej} are the average value of the potentials in elements e_i and e_j respectively.

As the flux can be described by using the magnetic potential, (2) may be rewritten as:

$$\psi = \sum_i w_{ei} A_{ei} - \sum_j w_{ej} A_{ej}. \tag{4}$$

Therefore, E can also be considered as the sum of the contributions of the elements in the armature coil region, that is

$$E = -\left(\sum_i w_{ei} \frac{dA_{ei}}{dt} - \sum_j w_{ej} \frac{dA_{ej}}{dt} \right). \tag{5}$$

Take the contribution of element $\boldsymbol{e}$ to analyze, and define

$$E = -\left(\sum_e E_{ei} - \sum_e E_{ej} \right), \tag{6}$$

and

$$E_{ei} = w_{ei} \frac{dA_{ei}}{dt}. \tag{7}$$

As the variation of A_{ei} is caused by the rotor rotation, the back-emf in (7) can be rewritten as,

$$E_{ei} = w_{ei} \frac{dA_{ei}}{d\theta} \frac{d\theta}{dt} = \omega w_{ei} \frac{dA_{ei}}{d\theta}, \tag{8}$$

where, ω is the angular speed of the rotor.

The relationship between the potential, rotor position and rotor speed is described clearly in (8). However, this equation relates to FE solutions, and is not readily applicable to calculate the back-emf directly due to the existence of the derivative term $dA_{ei}/d\theta$.

For realizing the computation, in generally, the derivative in (8) is replaced by a numerical derivative, that is, the difference of magnetic potential with respect to rotor position [1]–[3]. For calculating such a potential difference, several FE solutions have to be used. When only two solutions are used, the equation is,

$$E_{ei} = \omega w_{ei} \frac{A_{e2} - A_{e1}}{\Delta\theta}. \tag{9}$$

where, A_{ei2} and A_{ei1} are the two FE solutions at the given rotor position, and $\Delta\theta$ is the rotor position displacement pertinent to related these two solutions.

We define the voltage-constant of the motor as

$$K_v = E_{\max}/\omega. \tag{10}$$

From the viewpoint of FE analysis, the voltage constant K_v can also be considered as the sum of the contribution from each element in the winding area,

$$K_v = \max\left(\sum_e k_{Ve}\right), \tag{11}$$

where, K_{ve} is the contribution of the element e. From (8) and (10), it can be found that

$$K_{ve} = w_{ei}\left[\max\left(\frac{dA_{ei}}{d\theta}\right)\right]. \tag{12}$$

For calculating K_{ve}, the derivative in (12) can be replaced by the displacement of the rotor position, that is

$$K_{ve} = w_{ei}\{\max[(A_{e2} - A_{e1})/\Delta\theta]\}. \tag{13}$$

Therefore, in the computation of the voltage-constant, at least two FE solutions are also necessary for the given rotor position.

In the application of (9) and (13) to the computation of the back-emf and voltage-constant, the necessity of using two, or more, FE solutions causes many problems. Multiple FE solutions adjacent to the given position are required. The back-emf and voltage-constant cannot be calculated directly from one FE solution for the given position. This leads to an increase in the computer time. If we desire a better accuracy in computing derivatives, the FE solutions should be obtained by using meshes with very small displacement. However, such a computation will easily cause the loss of significant figures, and error in the results may thus augmented.

Searching for the optimal step length of the displacement $\Delta\theta$ in (9) and (13) and rearranging the mesh accordingly usually require professional judgements and experience in using FE analysis. This causes computing overhead and is extremely undesirable, if the FE analysis is to be used as a design optimization tool.

The new method introduced in the following section is developed from the consideration how to avoid the problems caused by the numerical derivative. The method needs only one FE solution to compute the back-emf.

II. Computing Back-EMF with Single-FE-Solution Method

In the FE analysis, the potential distribution is obtained from the solution of following equation,

$$[s_{ij}][A_j] = [p_i], \tag{14}$$

where, $[s_{ij}]$ is the FE stiffness matrix, $[A_j]$ is the node potential vector. The value of s_{ij} in the stiffness matrix is related to the following items:

1. FE mesh used to subdivide the machine region;

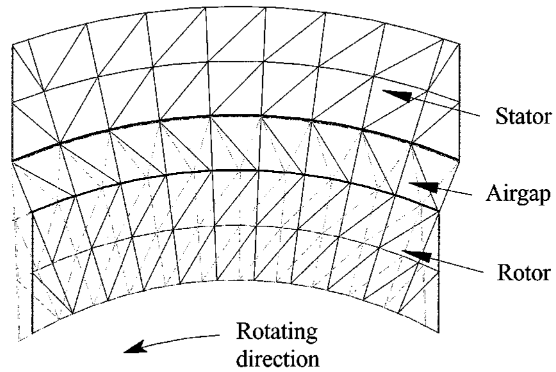

Fig. 1. Variation of airgap elements in the rotor moving (There is one layer of airgap elements. The elements before moving are the ones with solid lines).

2. The materials used in the machine; and
3. The relative position between the stator and rotor.

The second item makes the FE equation be nonlinear as the materials used in the stator and rotor of the machine are nonlinear in their magnetic characteristics. The last item shows that, $\boldsymbol{s}_{ij}$ is a function of rotor position and different FE meshes are needed to describe the FE relationship between the stator and rotor and in the rotor movement.

For the value p_i in the right hand vector, in addition to its relationship with the materials of the machine and FE mesh used, it is also the function of exciting current sources in the problem region.

After the rotor of the machine is rotated with a degree, the stator and rotor regions can still be described by using the same FE meshes. Only the coordinates of the nodes of the rotor mesh should be moved with the degree, but the airgap mesh must be adjusted, see Fig. 1. If such angular displacement is small and within a certain value, we can keep the airgap element topology, and change only the shape of the related elements in the airgap, see Fig. 1.

Calculating the derivative of both sides of (14) with respect to the rotor angular position, we have,

$$\left[\frac{ds_{ij}}{d\theta}\right][A_j] + [s_{ij}]\left[\frac{dA_j}{d\theta}\right] = \left[\frac{dp_i}{d\theta}\right]. \tag{15}$$

As there is no current in the airgap of the motor, p_i is independent of the airgap elements. Therefore, the movement of the rotor cannot affect the value of the right hand vector of (14). That means, $dp_i/d\theta$ is always zero. Equation (15) can thus be rewritten as

$$[s_{ij}]\left[\frac{dA_j}{d\theta}\right] = -\left[\frac{ds_{ij}}{d\theta}\right][A_j] = [b_i]. \tag{16}$$

As it has been discussed before, when the motor moves with a micro angle, we can analyze the effects of the movement with the same mesh topology. Therefore, when the motor areas have been subdivided by FE mesh, it is not difficult to calculate $ds_{ij}/d\theta$ from the expression of $\boldsymbol{s}_{ij}$. For simplifying the explanation, the following analysis will use the triangle element mesh in the analysis. For this kind of FE mesh, $\boldsymbol{s}_{ij}$ in the stiffness matrix of (14) and (16) are determined by [4], [5],

$$s_{ij} = s_{ji} = \frac{\nu}{4\Delta}(b_ib_j + c_ic_j) \tag{17}$$

where, ν is the reluctivity of the material in the element. b_i, b_j, c_i and c_j are the functions of the coordinates of the nodes [4], [5]. Δ is the area of the triangle, and it is certainly the function of the node coordinates.

For linear problems, the reluctivity ν in (17) is constant. However, as mentioned above, a slight movement of the rotor causes some airgap elements to change their shape and the nodal coordinates. These changes will certainly change the value of s_{ij}. The following equation can thus be obtained,

$$\frac{ds_{ij}}{d\theta} = \frac{\nu}{4}\left[-\frac{(b_ib_j + c_ic_j)}{\Delta^2}\,\frac{d\Delta}{d\theta} + \frac{d(b_ib_j + c_ic_j)/d\theta}{\Delta}\right]. \tag{18}$$

It is clear, when the expression for $\boldsymbol{s}_{ij}$ is given, it is not difficult to calculate its derivative $ds_{ij}/d\theta$. From this point of view, in (16), all the matrixes, except the vector $[A'_j]$, are known after the FE solution has been obtained. Therefore, the derivative $[A'_j]$ can be obtained by solving (16) directly.

For nonlinear problems, the reluctivity ν in (17) is not a constant. However, the following equation can be used to calculate the derivative,

$$\frac{ds_{ij}}{d\theta} = \frac{\nu}{4}\left[-\frac{(b_ib_j + c_ic_j)}{\Delta^2}\,\frac{d\Delta}{d\theta} + \frac{d(b_ib_j + c_ic_j)/d\theta}{\Delta}\right] + \frac{(b_ib_j + c_ic_j)}{4\Delta}\,\frac{d\nu}{d\theta}. \tag{19}$$

Consider that the reluctivity is a function of flux density in the element, the derivative $d\nu/d\theta$ can be described by

$$\frac{d\nu}{d\theta} = \frac{d\nu}{dB}\sum_l \frac{dB}{dA_l}\,\frac{dA_l}{d\theta} \tag{20}$$

where, B is the flux density in the element, A_l is the potential on the nodes of the element.

Therefore, for a nonlinear problem, the derivative $d\nu/d\theta$ is also a function of the derivative $dA/d\theta$, that is, the function of $[A'_j]$. This make the role of the magnetic potential more complicated. However, from (16) to (20), it is not difficult to obtain the following equation,

$$[s_{ij} + h_{ij}][A'_j] = [b_i], \tag{21}$$

where, h_{ij} is determined by the characteristic of nonlinear magnetic materials and the distribution of A_l. The right hand vector $[b_i]$ has the same meaning as in the cases of linear problems, see (16).

After the FE equation described by (14) has been solved, the potential distribution $[A_j]$ is obtained. The flux density in each element, and then the reluctivity of materials, can thus be known. All these show that, we can build up the matrixes $[s_{ij} + h_{ij}]$ and $[b_i]$ after obtaining FE solution. Therefore, the derivative $\partial A_i/\partial\theta$ in the nonlinear problems can still be obtained through solving (21), an algebraic system of equations, in the post processing stage of FE analysis.

As the derivative $\partial A_i/\partial\theta$ is known, the back-emf of the armature windings and the voltage-constant of the motor can be readily computed by using (8) and (12) respectively. In such computations, for every given rotor position, calculating back-emf requires only a single FE solution. In the following sections, such a computation method will be called Single-FE-Solution-Method, and SSM for short.

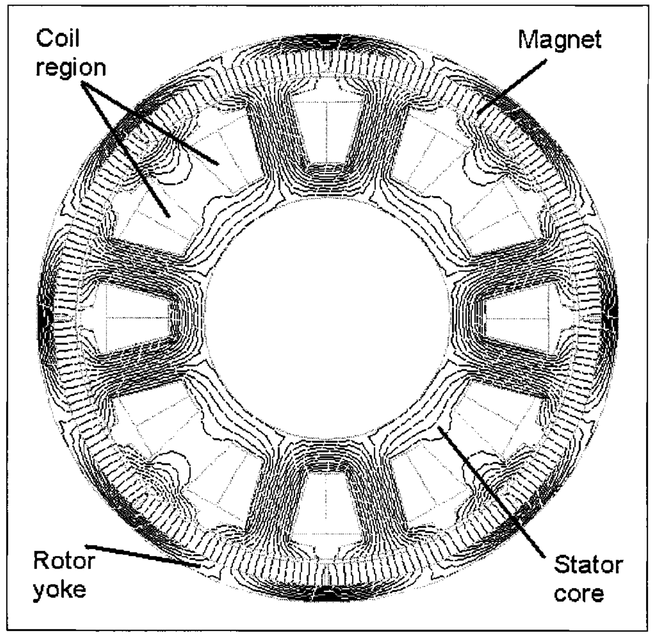

Fig. 2. Field distribution in the motor.

III. The Application of SSM

The authors have used SSM in the design and analysis of different kinds of PM BLDC and synchronous motors. These applications show that SSM is effective in the estimation of back-emf and voltage-constant of the motors. In this section, one of these applications is described in details to express the effects of the proposed method.

Consider a spindle motor used in a 3.5″ hard disk drive. This is a 3-Phase PM BLDC motor with outer rotor. There are 12 stator slots and four pole-pairs. The field distribution of the motor is also shown in Fig. 2, which is obtained by using FE method.

In the FE analysis for this spindle motor, the triangle elements are used. In the computation, the flux linkage in one element is processed as

$$\psi_e = \frac{w_s}{3}\,\frac{\Delta_e}{\Omega_w}(A_i + A_j + A_m) \tag{22}$$

where, Δ_e is the element area, Ω_w is the winding area in one slot, and w_s is the turns of one phase winding in a slot. A_i, A_j and A_m are the potential values of the three element nodes.

The SSM is used to compute the back-emf of the armature windings generated in the motor operation with speed 5400 rpm, and the results are shown in Fig. 3.

From the analysis in Section II, SSM can be used to calculate the back-emf produced in many kinds of electric machines. The reason for taking the spindle motor as example here is due to the readiness of the test data to check the effects of the analysis result. Fig. 4 shows the testing result at 5400 rpm. Filtering the EM noise produced in the data sampling and comparing with the computation result obtained with SSM which is shown in Fig. 3, these results agree well.

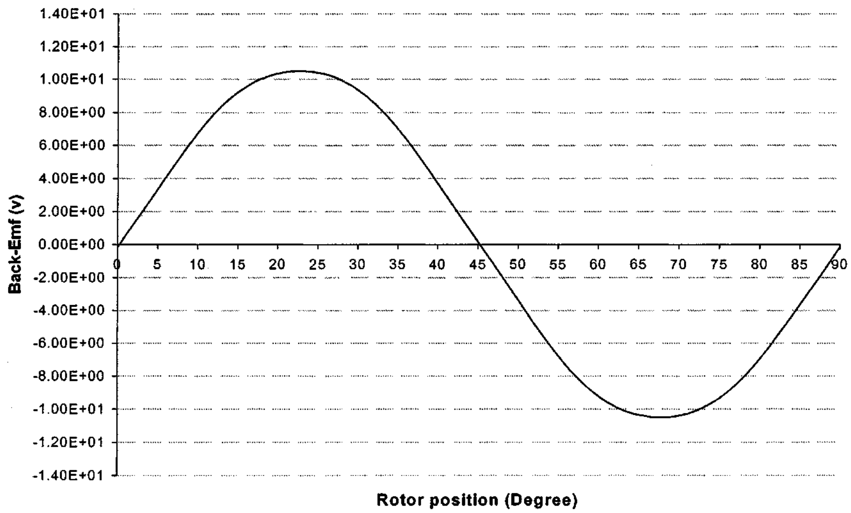

Fig. 3. The back-emf estimated by using SSM (Speed: 5400 rpm).

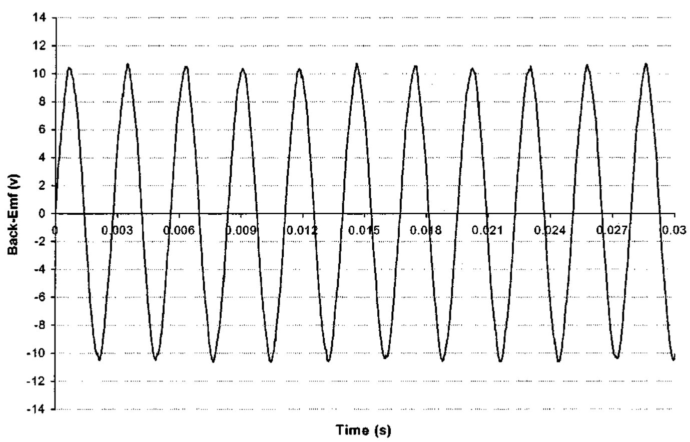

Fig. 4. Testing results of the Back-Emf generated at 5400 rpm.

The Voltage-Constant of the motor is also calculated by using SSM and experimental measurements, as given in Table I.

In the research, the authors applied also the Numerical Differentiation Approach as described by (9) to calculate the back-emf. In order to achieve better accuracy, a "half-step forward, and half-step backward" strategy is used, and the step length $\Delta\theta$ shown in (9) is adjusted. It was found, when $\Delta\theta$ is reduced less than 2 degree but larger than 0.2 degree, the results converge, and they are very close to that obtained from SSM. This confirms also the effectiveness of the SSM in back-emf estimation.

IV. Conclusions

The back-emf and voltage-constant of PM motors can be estimated by using the numerical differentiation approach in the post processing of FE analysis. However, such approach suffered many drawbacks. Firstly, multiple FE solutions for one rotor position are required, resulting in higher cost in computer time consumption. The successfulness of the computational process relies heavily on the user's judgement and experience. Secondly, since these solutions are computed on the different but very similar domain and boundary conditions, the process

TABLE I
Voltage-Constant of the Spindle Motor

Method	Voltage Constant
SSM	0.00194 (v/rpm)
Test	0.00198 (v/rpm)

is prone to numerical errors. There may be a considerable loss of significant figures in the computation. On the other hand, it is apparent that large step length will lead to poor accuracy in the derivative computation as described in (9) and (13).

The method presented in the paper utilizes only one FE solution to compute the back-emf and voltage-constant at the given rotor position. It uses the derivative of the potential directly in the computation. The problems caused by using the numerical differentiation can be avoided.

In the paper, an algorithm for calculating the derivative of the magnetic potential with respect to rotor position is introduced. The algorithm can calculate the derivative conveniently in the post processing of FE analysis.

Both theoretical analysis and numerical computation show the effectiveness of SSM. It is effective for both linear and nonlinear problems.

Though the analysis of the paper focuses on the PM BLDC motors, it is apparent that, SSM is also applicable in the design and analysis of other kinds of electric machines.

References

[1] P. Lombard and G. Meunier, "General method for electric and magnetic coupled problem in 2D and magnetodynamic domain," *IEEE Trans. on Magnetics*, vol. 28, no. 2, pp. 1291–1294, Mar. 1992.
[2] N. Mieko, J. R. Cardoso, and A. Foggia, "Coupling electric and 2D-FEM model with dommel's approach for transient analysis," *IEEE Trans. on Magnetics*, vol. 34, no. 5, pp. 3487–3490, Sept. 1998.
[3] F. Caricchi, F. Crescimbini, E. Santini, and C. Santucci, FEM evaluation of performance of axial flux slotted permanent magnet machines, in 33rd IAS Annual Meeting, vol. 1, pp. 12–17, 1998.
[4] P. P. Silvester and R. L. Ferrari, *Finite Elements for Electrical Engineers*: Cambridge University Press, 1990.
[5] S. J. Salon, *Finite Element Analysis of Electrical Machines*: Kluwer Academic Publishers, 1995.