Motor "Identity"—A Motor Model for Torque Analysis and Control

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***Abstract*— This paper introduces a novel concept in torque analysis and torque controller design, using *motor identity*. The concept of identity is defined and analysis for generating an optimal input current which can produce smooth and maximum torque output is described. The extraction process of identity from a motor is also provided. Simulations and experiments applying this identity concept in torque control on a permanent magnet synchronous motor are given. The results are compared with a conventional motor controller using 3-phase sinusoidal currents.**

I. Introduction

PRODUCTION of smooth and maximum torque are two main performance goals in torque control of permanent magnet synchronous motors (PMSM's). Various proposals have been advanced to solve these problems [1]–[3]. In all these proposals motor parameters such as inductances and resistances are used in lumped parameter models and used with the proposed control strategy with a view to achieve good torque performance.

This paper presents a new approach for modeling the motor for torque analysis and control. This method is unconventional in that no parametric model of the motor is required, instead a new concept of motor identity is introduced and used. This identity can be represented in a matrix form—called the identity matrix. This characterizes the motor and this matrix can be used for torque analysis and control. The concept of identity is developed in order to provide an approach for determining the optimal current profile to achieve a specified torque performance. This concept may also be applicable to other synchronous machines but a PMSM is used here for the illustration of this concept. An analysis is also presented for deriving an optimal drive currents, which produces maximum and smooth torque, by using the motor identity. A method to extract the motor identity experimentally is also given in this paper.

II. Torque Production in PMSM's

In the linear state where the machine is not saturated, the composite electromagnetic torque T_s in a PMSM is described as [4]:

$$T_s = \frac{1}{2}[I_t] \cdot \frac{dL}{d\theta} \cdot [I_t]^T \tag{1}$$

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where $[I_t]$ is the phase current matrix and L is the inductance matrix of the motor.

A conventional 3-phase synchronous machine contains four circuits. The armature coils A, B, C and field exciting coil f. Therefore,

$$[I_t] = [i_f \quad i_a \quad i_b \quad i_c] \tag{2}$$

$$L = \begin{bmatrix} L_f & M_{fa} & M_{fb} & M_{fa} \\ M_{af} & L_a & M_{ab} & M_{ac} \\ M_{bf} & M_{ba} & L_b & M_{bc} \\ M_{cf} & M_{ca} & M_{cb} & L_c \end{bmatrix} \tag{3}$$

where L_i is the self inductance and M_{ii} the mutual inductance. The subscripts f, a, b, c denote the field coil, A-phase armature coil, B-phase armature coil and C-phase armature coil, respectively.

It is easy to prove using field theory that if linearity is assumed, $M_{xy} = M_{yx}$, where x and y are dummy variables for the matrix above. The experimental results reported in reference [1] pointed out that this equation also holds well in the saturated state. Therefore, (1) can be rewritten as

$$\begin{aligned} T_s = &\frac{1}{2}\left(i_f^2 \frac{dL_f}{d\theta} + i_a^2 \frac{dL_a}{d\theta} + i_b^2 \frac{dL_b}{d\theta} + i_c^2 \frac{dL_c}{d\theta}\right) \\ &+ i_f i_a \frac{dM_{fa}}{d\theta} + i_f i_b \frac{dM_{fb}}{d\theta} + i_f i_a \frac{dM_{fc}}{d\theta} \\ &+ i_a i_b \frac{dM_{ab}}{d\theta} + i_a i_c \frac{dM_{ac}}{d\theta} + i_b i_c \frac{dM_{bc}}{d\theta}. \end{aligned} \tag{4}$$

The first term on the right-hand side of (4) expresses a torque component which is due to the variation of self-reluctance of the field coil. In the following analysis, this component will be denoted by T_f, that is

$$T_f = \frac{1}{2} i_f^2 \frac{dL_f}{d\theta}. \tag{5}$$

The variation of self reluctance of the field coil is due to the existence of stator slots. The self reluctance of the field coil varies depending on the rotor position. This variation produces the torque T_f as the field produced by the field current tends to minimize the self reluctance of the field coil. This cogging torque is a reluctance torque and it is not related to armature currents.

Cogging torque exists as an electromagnetic torque in synchronous machines. However, cogging torques can be easily eliminated with skewed slots or skewed poles in the PMSM [5]. For simplicity, the cogging torque component is neglected in this study [1]. With this assumption the torque equation (4) can be rewritten as

$$\begin{aligned}T_s = &\frac{1}{2}\left(i_a^2\frac{dL_a}{d\theta} + i_b^2\frac{dL_b}{d\theta} + i_c^2\frac{dL_c}{d\theta}\right)\\ &+ i_f i_a\frac{dM_{fa}}{d\theta} + i_f i_b\frac{dM_{fb}}{d\theta} + i_f i_a\frac{dM_{fc}}{d\theta}\\ &+ i_a i_b\frac{dM_{ab}}{d\theta} + i_a i_c\frac{dM_{ac}}{d\theta} + i_b i_c\frac{dM_{bc}}{d\theta}. \end{aligned}\tag{6}$$

III. Torque Components

The composite output torque of a motor can also be expressed as a sum of the torque produced by the individual phases, called the "component torques." This concept of component torque is used here to facilitate the understanding of the concept of motor identity. Using this concept, (6) is can be rewritten as

$$T_s(\theta) = T_A(\theta) + T_B(\theta) + T_C(\theta) \tag{7}$$

where

$$T_A = i_f i_a\frac{dM_{fa}}{d\theta} + \frac{1}{2}\left(i_a^2\frac{dL_a}{d\theta} + i_a i_b\frac{dM_{ab}}{d\theta} + i_a i_c\frac{dM_{ac}}{d\theta}\right) \tag{8}$$

$$T_B = i_f i_b\frac{dM_{fb}}{d\theta} + \frac{1}{2}\left(i_b^2\frac{dL_b}{d\theta} + i_b i_a\frac{dM_{ab}}{d\theta} + i_b i_c\frac{dM_{bc}}{d\theta}\right) \tag{9}$$

$$T_C = i_f i_c\frac{dM_{fc}}{d\theta} + \frac{1}{2}\left(i_c^2\frac{dL_c}{d\theta} + i_b i_c\frac{dM_{bc}}{d\theta} + i_a i_c\frac{dM_{ac}}{d\theta}\right). \tag{10}$$

In this way, the electromagntic torque in a synchronous machine can be described by the component torques T_A, T_B and T_C. These torque components are defined as A-component torque, B-component torque and C-component torque, respectively.

For a synchronous motor which has a symmetrical structure, its three sequence torques are symmetrical. Therefore, in the linear state, the analysis for one sequence torque allows the other sequence torque and hence the composite torque of the motor to be deduced.

The A-component torque can be further expressed as

$$T_A(\theta) = T_{AF}(\theta) + T_{AA}(\theta) + T_{AB}(\theta) + T_{AC}(\theta) \tag{11}$$

where

$$T_{af}(\theta) = i_a i_f\frac{dM_{af}}{d\theta} \tag{12}$$

$$T_{aa}(\theta) = \frac{1}{2}i_a^2\frac{dL_a}{d\theta} \tag{13}$$

$$T_{ab}(\theta) = \frac{1}{2}i_a i_b\frac{dM_{ab}}{d\theta} \tag{14}$$

$$T_{ac}(\theta) = \frac{1}{2}i_a i_c\frac{dM_{ac}}{d\theta}. \tag{15}$$

T_{af} in (12) is due to the interaction between the A-phase current and PM exciting field. T_{aa} of (13) is produced by the variation of self-reluctance of the A-phase armature coil, T_{ab} of (14) and T_{ac} of (15) are the results of the variation of mutual reluctance between A and B phase armature coils, respectively. The first T_{af} is conventionally known as excitation torque component and the latter three components are reluctance torque components.

IV. The Identity of a PMSM

Equation (11) can be rewritten as

$$\begin{aligned}T_A &= i_a\cdot[i_f T_{af} + i_a T_{aa} + i_b T_{ab} + i_c T_{ac}]\\ &= i_a\cdot[T_{af}\quad [T_{aa}\quad T_{ab}\quad T_{ac}]\cdot[i_f\quad i_a\quad i_b\quad i_c]^T\\ &= i_a\cdot[T_{af}\quad [R_a]]\cdot[I_t]^T\\ &= i_a\cdot[A]\cdot[I_t]^T.\end{aligned}\tag{16}$$

$[R_a]$ is defined as the A-component reluctance torque matrix. The matrix $[A]$ in (16) is defined as the A-component Identity Matrix, or the "identity" of the motor.

In a PMSM, the exciting field produced by the PM material can be represented as a field produced by a constant field current. When this field current is defined as the base field current $i_{fb}, [I_t]$, and $[A]$ can be rewritten as

$$\begin{aligned}[I_t] &= [1\quad i_a\quad i_b\quad i_c]\\ [A] &= [T_{af}\quad [R_a]].\end{aligned}\tag{17}$$

From (16), the A-component reluctance matrix $[R_a]$ is

$$[R_a] = [T_{aa}\quad T_{ab}\quad T_{ac}] \tag{18}$$

In a symmetrical three-phase PMSM

$$T_A(\theta - 240^\circ) = T_B(\theta - 120^\circ) = T_C(\theta). \tag{19}$$

Therefore, (7) can be rewritten as

$$T_s(\theta) = T_A(\theta) + T_A(\theta - 120^\circ) + T_A(\theta - 240^\circ). \tag{20}$$

Equation (20) shows that if the A-component identity matrix, or the motor identity, is known, the torque-position characteristic of the PMSM can be deduced from the matrix for any drive currents. The use of motor identity thus provides a facility for torque analysis of the PMSM.

The self inductances and mutual inductances of the electrical machines can be expressed with Fourier series. When is chosen as the angle between axes fixed to the moving field coil on the rotor and the stationary A phase armature coil on the stator

$$L_c(\theta) = L_b(\theta - 120^\circ) = L_a(\theta - 240^\circ) \tag{21}$$

$$M_{ca}(\theta) = M_{bc}(\theta - 120^\circ = M_{ab}(\theta - 240^\circ) \tag{22}$$

$$M_{fc}(\theta) = M_{fb}(\theta - 120^\circ) = M_{fa}(\theta - 240^\circ) \tag{23}$$

and

$$M_{af} = \sum_{n=1}^{\infty} F_n\cos[(2n-1)\theta] \tag{24}$$

$$L_a = A_o + \sum_{n=1}^{\infty} A_n\cos(2n\theta) \tag{25}$$

$$M_{ab} = B_o + \sum_{n=1}^{\infty} B_n\cos[2n(\theta - 60^\circ)] \tag{26}$$

$$M_{ac} = C_o + \sum_{n=1}^{\infty} C_n\cos[2n(\theta + 60^\circ)]. \tag{27}$$

With these expressions, the components of the motor identity, as in (12) to (15), will be

$$T_{af}(\theta) = \sum_{n=1}^{\infty} T_{afn} \sin[(2n-1)\theta] \tag{28}$$

$$T_{aa}(\theta) = \sum_{n=1}^{\infty} T_{aan} \sin(2n\theta) \tag{29}$$

$$T_{ab}(\theta) = \sum_{n=1}^{\infty} T_{abn} \sin[2n(\theta - 60^\circ)] \tag{30}$$

$$T_{ac}(\theta) = \sum_{n=1}^{\infty} T_{acn} \sin[2n(\theta + 60^\circ)]. \tag{31}$$

The above equations for the torque components fit in the expression of identity in (17)

$$T_A = i_a \cdot [i_f T_{af} + i_a T_{aa} + i_b T_{ab} + i_c T_{ac}] \tag{32}$$

to give the motor identity. The identity of the motor is thus a complicated function of the rotor position.

However, when the effects of the higher order harmonics of these torque components are neglected, the A-Sequence Identity Matrix is simplified to

$$\begin{aligned} A(\theta) = [&T_{af1}\sin(\theta) \quad T_{aa1}\sin(2\theta) \quad T_{ab1}\sin(2\theta - 120^\circ) \\ &T_{ac1}\sin(2\theta + 120^\circ)]. \end{aligned} \tag{33}$$

The identity that is characterized by this matrix is a simplified one, and a PMSM model which uses only such fundamental terms or first-order harmonics is termed the "first-order model" of the PMSM. It retains the important characteristics of the PMSM as the fundamental terms of identity are used. This simplified motor will be used to illustrate the use of motor identity in torque analysis and control and it is used in later sections.

V. Determination of Optimal Drive Currents Using Motor Identity

In motor drive control, there are always two desired performance specifications on drive currents: one which satisfies requirements to produce torque with no ripples, and another one to produce maximum torque for a given prespecified root-mean-square value of drive current.

The component torque, and hence the total composite torque produced in a PMSM, have been shown to be functions of input currents and the motor identity, as in (16). The determination of the ideal excitation that will produce a maximum per-phase torque or smooth torque while considering all the harmonics in current and identity can thus be formidable. In this paper, a drive current which satisfies both the smooth torque and maximum torque requirements is defined here as the *Optimal Drive Current.* An attempt is made here to determine this optimal current for PMSM's, based on the concept of motor identity.

A spectral analysis on the motor identity can yield useful results on the production of smooth torque. The A-component torque of a PMSM can be expressed as a Fourier series

$$T_A(\theta) = \sum_{n=0}^{N} T_{An} \cos(n\theta + \beta_n). \tag{34}$$

The components of this series can be classified into four types of harmonics

Type 1: $n = 0$;
Type 2: $n = 3i$;
Type 3: $n = 3i + 1$;
Type 4: $n = 3i - 1$;

where $i = 1, 2, 3, \cdots$.

Due to the symmetry in machine structure, the types 3 and 4 harmonics do not contribute to the EM torque production of the PMSM. Therefore, substituting (34) to (20)

$$\begin{aligned} T_s(\theta) &= T_A(\theta) + T_A(\theta - 120^\circ) + T_A(\theta - 240^\circ) \\ &= 3T_{Ao} + 3\sum_{i=1}^{\infty} T_{A3i} \cos[3i(\theta - \xi_{3i})] \end{aligned} \tag{35}$$

where T_{A0} denotes a dc component with zero-order harmonics. Equation (35) clearly shows that *if a constant and smooth torque is to be obtained, the triplen harmonics must not exist in the component torque* T_A. Phase current should thus be designed to satisfy this requirement. Following this requirement, however, it should be reminded that phase current may contain triplen harmonics and arrangement has to be made to allow triplen harmonics to flow in the motor.

Maximization of the torque production isuivalent to the minimization of losses from the drive current. To assess the copper loss of a motor, the index p_1 described by (36) is used

$$p_1 = \frac{1}{T} \int_0^T i_a^2(t)\, dt \tag{36}$$

where T is the period of the drive current.

If the motor is running in the steady-state mode and taking the rotor position as a function of time t, (36) can be written as

$$p_1 = \frac{1}{2\pi} \int_0^{2\pi} i_a^2(\theta)\, d\theta. \tag{37}$$

Hence, a desirable current profile $i_a(\theta)$ that can fullfil the maximum torque requirement can be stated as a current that satisfies the optimization of p_1 in (37). Equation (35) has shown that the triplen harmonics should not exist for a smooth torque production. As such the problem of finding the optimal drive current is equivalent to the following optimization problem:

$$\begin{aligned} &\text{Minimize } \left[\left[\int_0^{2\pi} i_a^2(\theta)\, d\theta\right]\right]\Big|_{T_{A0}=\text{const}} \\ &\int_0^{2\pi} T_A(i_a, \theta) \sin(3n\theta)\, d\theta = 0 \\ &\int_0^{2\pi} T_A(i_a, \theta) \cos(3n\theta)\, d\theta = 0 \quad \text{with } n = 1, 2, 3 \cdots. \end{aligned} \tag{38}$$

The last two equations in the optimization are requirement for the production of smooth torque in PMSM, as described earlier. The problem can be reformulated as

$$\text{Maximize } [T_{A0}|_{T_{A0}=\text{const}}]$$
$$\int_0^{2\pi} T_A(i_a,\theta)\sin(3n\theta)\,d\theta = 0$$
$$\int_0^{2\pi} T_A(i_a,\theta)\cos(3n\theta)\,d\theta = 0 \quad \text{with } n = 1,2,3\cdots. \tag{39}$$

The constrained variational problem described above is generally difficult to solve. However, for a PMSM having the simplified identity matrix A in (33), the optimal drive current can be found. This is described in the next section.

VI. The Optimal Drive Current for First-Order Model of a PMSM

A simplified model for a PMSM can be constructed by considering only the first-order harmonics in the A-component Identity Matrix, as already shown in (33). This model is termed the "first-order model" of the PMSM. In this section the optimal drive current will be synthezied for a PMSM represented by this model to illustrate the application of motor identity in torque control of PMSM's.

Optimization of (38) or (39) gives the solution of an optimal current. For the production of smooth torque, it has already been shown in last section that no triplen harmonics should exist in the sequence torque. As for the production of maximum possible developed torque, the copper loss incurred in a motor drive system is considered, as described in (36). The drive current for a PMSM can generally be expressed in Fourier Series as

$$\begin{aligned} i_a(\theta) &= \sum_{n=0}^{\infty} I_n \sin(n\theta + \xi_n) \\ i_b(\theta) &= i_a(\theta - 120^\circ) \\ i_c(\theta) &= i_a(\theta - 240^\circ). \end{aligned} \tag{40}$$

The copper loss is

$$p_1 = \tfrac{1}{2}\sum_{n=0}^{\infty} I_n^2. \tag{41}$$

Substituting (40) for the i_a of (16), and using only the fundamental terms in the motor identity, yields

$$T_A(\theta) = T_{AF}(\theta) + T_{AA}(\theta) + T_{AB}(\theta) + T_{AC}(\theta) \tag{42}$$

where

$$T_{AF}(\theta) = T_{af1}\sin(\theta)\cdot\sum_{n=0}^{N} I_n \sin(n\theta + \xi_n) \tag{43}$$

$$T_{AA}(\theta) = T_{aa1}\sin(2\theta)\cdot\left[\sum_{n=0}^{N} I_n \sin(n\theta + \xi_n)\right]^2 \tag{44}$$

$$T_{AB}(\theta) = T_{ab1}\sin(2\theta - 120^\circ)\cdot\left[\sum_{n=0}^{N} I_n \sin(n\theta + \xi_n)\right]$$
$$\cdot\left[\sum_{n=0}^{N} I_n \sin[n(\theta) - 120^\circ) + \xi_n]\right] \tag{45}$$

$$T_{AC}(\theta) = T_{ac1}\sin(2\theta + 120^\circ)\cdot\left[\sum_{n=0}^{N} I_n \sin(n\theta + \xi_n)\right]$$
$$\cdot\left[\sum_{n=0}^{N} I_n \sin[n(\theta + 120^\circ) + \xi_n]\right]. \tag{46}$$

$$\begin{aligned} T_{af} &= 1.928\sin(\theta) + 0.28\sin(3\theta) - 0.06\sin(5\theta) \\ T_{aa} &= 0.556\sin(2\theta) - 0.09\sin(6\theta) + 0.041\sin(10\theta) \\ T_{ab} &= 0.26\sin[2(\theta - 60^\circ)] - 0.043\sin[6(\theta - 60^\circ)] + 0.018\sin[10(\theta - 60^\circ)] \\ T_{ac}(\theta) &= T_{ab}(\theta + 120^\circ) \end{aligned}$$

Fig. 1. Motor identity for experimental motor.

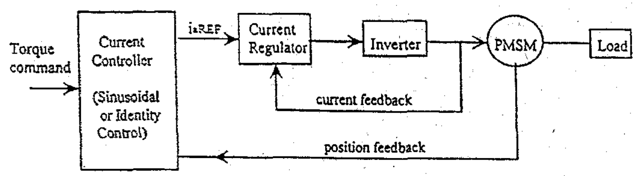


Fig. 2. Torque controller based on motor identity.

In the above equation, N can be any integer. If N is greater than 1, torque ripples will be produced. Therefore, an *optimal* drive current for the first-order PMSM is a sinusoidal current with

$$\begin{aligned} i_a &= I\cdot\sin(\theta + \xi) \\ i_b &= I\cdot\sin(\theta - 120^\circ + \xi) \\ i_c &= I\cdot\sin(\theta + 120^\circ + \xi). \end{aligned} \tag{47}$$

With the sinusoidal current excitation chosen, the constrained variational problem of (39) is modified to a constrained one,

$$\text{Maximize } T_{A0} \quad p_{1=\text{constant}}. \tag{48}$$

The expression for p_1 is shown in (37). As $T_{ab1} = T_{ac1}$, we have

$$T_{A0} = \frac{1}{2}\cdot T_{af1}\cdot I\cos(\xi) + \frac{I^2}{4}\cdot(T_{aa1} + 2T_{ab1})\cdot\sin(2\xi). \tag{49}$$

To maximize the torque T_{A0}, (49) is differentiated with respect to ξ and the derivative set to zero

$$\frac{d}{d\xi}T_{A0} = -\frac{1}{2}\cdot T_{af1}\cdot I\cdot\sin(\xi) + \frac{I^2}{2}\cdot(T_{aa1} + 2T_{ab1})\cdot\cos(2\xi). \tag{50}$$

Therefore

$$\sin(\xi) = \frac{-T_{af1} \pm \sqrt{T_{af1}^2 + 8I^2(T_{aa1} + 2T_{ab1})^2}}{4I(T_{aa1} + 2T_{ab1})} \tag{51}$$

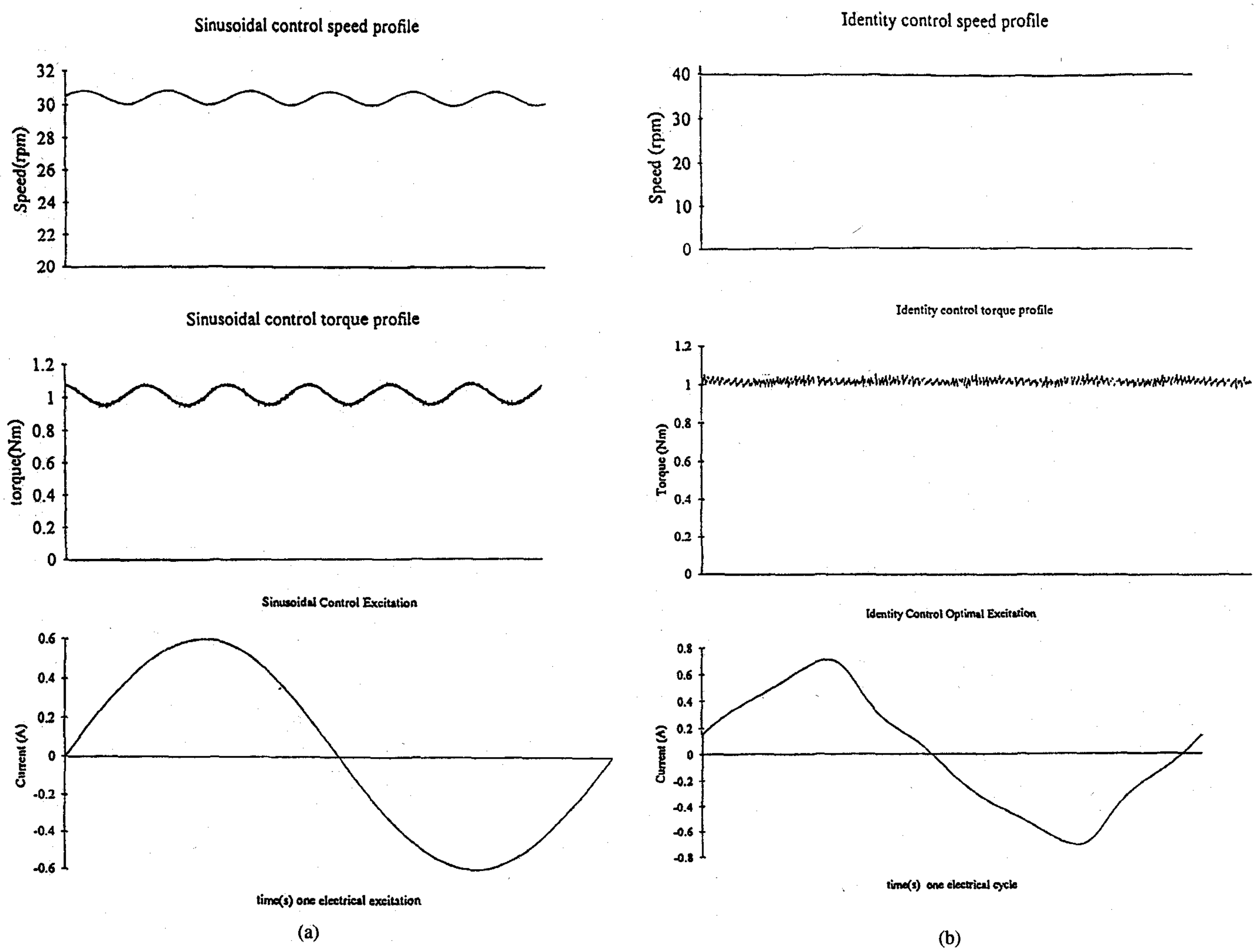


Fig. 3. Simulation results. (a) Sinusoidal current control. (b) Identity current control.

and solution for the equation (51) gives

$$\xi = \begin{cases} \sin^{-1}\left(\dfrac{-T_{af1} + \sqrt{T_{af1}^2 + 8I^2(T_{aa1} + 2T_{ab1})^2}}{4I(T_{aa1} + 2T_{ab1})}\right) \\ \text{for } T_{af_1} \geq 0 \\ \sin^{-1}\left(\dfrac{-T_{af1} - \sqrt{T_{af1}^2 + 8I^2(T_{aa1} + 2T_{ab1})^2}}{4I(T_{aa1} + 2T_{ab1})}\right) \\ \text{for } T_{af1} < 0 \end{cases} \tag{52}$$

The solution of a delay angle ξ provides the optimal current for driving the PMSM. This is a sinusoidal current with the phase angle determined by (52). This result can be further generalized to motors having arbitrary identity, which may require more elaborate derivation and computation. Similar results for inserting a delay angle to achieve maximum torque performance have been obtained by other authors using analytical approaches based on dq theory [9].

In summary, the optimal phase current should possess two features: first, it should not cause any triplen harmonics in the torque components; and secondly, a phase angle, which can determined by the identity of the motor, should be used in the current.

In this example, the PMSM is assumed to be a motor with the identity consisting of only the first-order components. In general, the higher order components of the identity exist. In such cases the optimal drive current will have higher order harmonics.

VII. Extraction of Motor Identity

The identity of a PMSM can of course be extracted from the measurements of the mutual inductances $M_{af}(\theta), M_{ab}(\theta)$ and self inductances $L_a(\theta)$, as suggested in (12)–(15). The measurements for these inductances can be done using either ac or dc methods [4]. However, identity has to be computed using derivations and this reduces the accuracy of the results. Another problem of such indirect measurements is that it is hard to control the position of the rotor accurately.

A method is described below for the direct measurement of the motor identity. The voltage equation for A-phase coil is

$$U_a = \frac{d\Psi_{af}}{dt} + \frac{d\Psi_{aa}}{dt} + \frac{d\Psi_{ab}}{dt} + i_a r_a \tag{53}$$

where $\Psi_{af}, \Psi_{aa}, \Psi_{ab}$ and Ψ_{ac} are the flux linkages of A-phase coil induced by i_f, i_a, i_b and i_c respectively. When the motor is driven to rotate at a constant speed ω and the armature

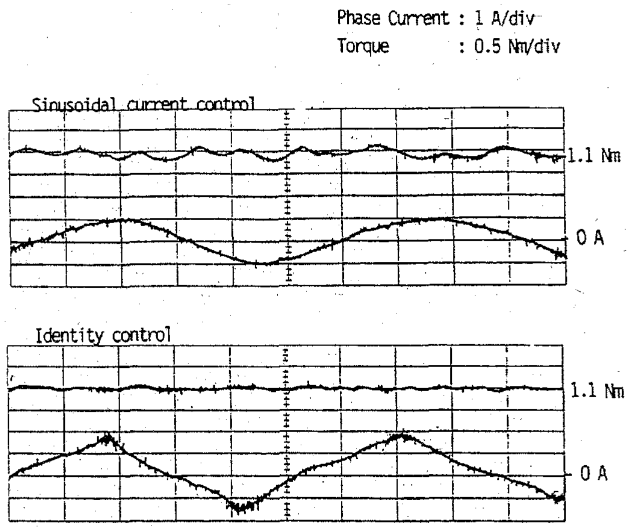


Fig. 4. Experimental results.

current are kept constant. This equation can be rewritten as

$$U_a = \omega\left[i_f\frac{dM_{af}}{d\theta} + i_a\frac{dL_a}{d\theta} + i_b\frac{dM_{ab}}{d\theta} + i_f\frac{dM_{ac}}{d\theta}\right] + i_a r_a. \tag{54}$$

Based on this equation, the motor identity can be derived by using the following steps.

Step 1: The motor is driven at a constant speed ω. Let i_a, i_b and i_c be zero. Record A-phase voltage $U_{a1}(\theta)$. It yields

$$T_{af}(\theta) = \frac{1}{\omega}\cdot U_{a1}(\theta). \tag{55}$$

Step 2: Let $i_a = i_{\text{base}}, i_b$ and i_c be zero, record A-phase voltage $U_{a2}(\theta)$, B-phase voltage $U_{b2}(\theta)$, and C-phase voltage $U_{c2}(\theta)$.

Step 3: Let $i_a = -i_{\text{base}}, i_b$ and i_c be zero, record A-phase voltage $U_{a3}(\theta)$, B-phase voltage $U_{b3}(\theta)$, and C-phase voltage $U_{c3}(\theta)$.

Step 4: Calculate the components $T_{aa}(\theta), T_{ab}(\theta), T_{ac}(\theta)$ with the following equations.

$$\begin{aligned} T_{aa} &= \frac{1}{2\omega}[U_{a2}(\theta) - U_{a3}(\theta) - 2i_a r_a] \\ T_{ab} &= \frac{1}{2\omega}[U_{b2}(\theta) - U_{b3}(\theta) - 2i_a r_a] \\ T_{ac} &= \frac{1}{2\omega}[U_{c2}(\theta) - U_{c3}(\theta) - 2i_a r_a]. \end{aligned} \tag{56}$$

With these voltage acquisitions and computations, all the components of the identity can be obtained as in (17)–(18).

VIII. Simulation and Experimental Results

Simulation and experiments have been carried out on a PMSM with multistacked imbricated rotor design [8] to prove the feasibility of using motor identity in torque control. The identity of the motor is experimentally acquired and is as shown in Fig. 1.

The simulations and experiments are based on the controller structure depicted in Fig. 2, where a specified torque reference command is fed to the identity controller, and the identity algorithm generates an optimal current that ensures both smoothness and maximum developed torque. The results are compared with a conventional sinuosisal current control, also known as $i_d = 0$ control [9], where sinusoidal currents are fed to the motor.

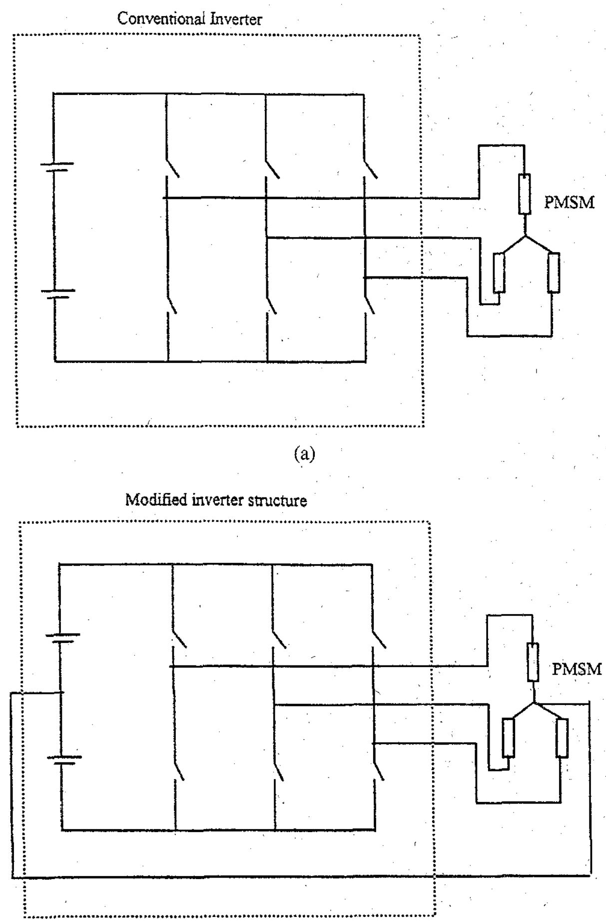


Fig. 5. Converter structure for identity controller.

Extensive simulations and experiments have been carried out and a typical simulation result is shown in Fig. 3. An experimental result is also shown in Fig. 4. The results show an improvement on the torque ripples and the magnitude of torque that can be produced by the same rms current.

It is worthwhile to point out an essential point in the implementation of torque controller based on identity analysis: there are triplen harmonics in the system and a new covnerter structure in Fig. 5 has to be employed to allow these harmonics to flow in the system. The harmonics in a typical optimal current is shown in Fig. 6.

IX. Discussions

From the analysis of identity control provided in this paper, an optimal input phase current can be determined based on the motor identity, to produce optimum torque performance. However, if a desired identity is specified, it is also possible to optimize the motor structure to give the desired motor identity.

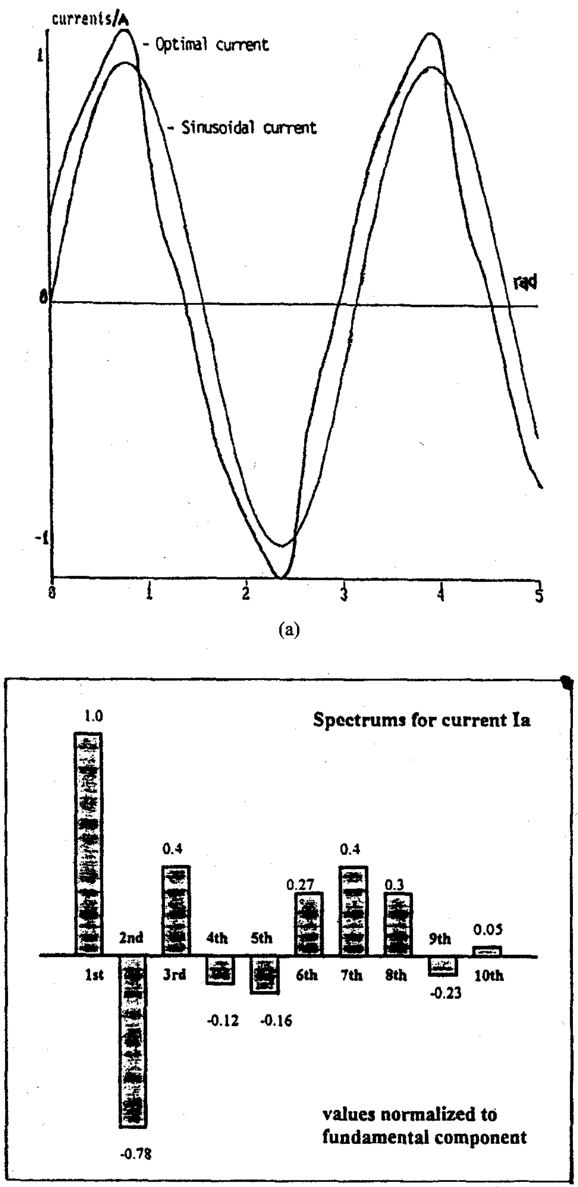


Fig. 6. Harmonic contents in a typical optimal current.

The first problem of finding an optimum current to control the PMSM is termed as the Identity Control. The second problem of designing a motor structure to achieve a specified identity, is termed the Identity Design, and it falls into the category of inverse electromagnetics [6], [7]. To illustrate

Identity Control—For drive controller designs;
—Identity of the motor is experimentally acquired, controller is then designed and optimized to achieve optimal torque performances.

Identity Design —For motor designs;
—Motor structure is optimized during the design process so that it will produce the desired motor identity.

The use of the optimal current described in this paper is limited by the power electronic devices used in the motor driver. In order to attain certain torque performance, current with high-frequency components may be required and this may well beyond the bandwidth of the power electronic systems.

X. Conclusions and Future Research

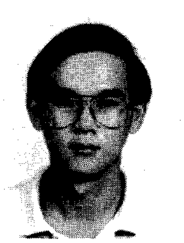
The concept of motor identity is introduced in this paper. The use of the identity in torque analsysis and torque control is also presented. It is shown that with the motor identity, an optimal drive current can be determined which can give smooth and maximum developed torque. A description of a direct measurement method to extract the identity from a PMSM is provided. Simulation and experimental results are also given in the paper. Research in extending the concept of identity to other synchronous motors will be presented in the future.

References

[1] J. S. Ko, J. H. Lee, and M. J. Youn, "Robust digital position control of brushless DC motor with adaptive load torque observer," in *IEE Proc. Elec. Power Applicat.*, vol. 141, no. 2, Mar., 1994, pp. 63–70.
[2] S. Morimoto, Y. Takeda, K. Hatanaka, Y. Tong, and T. Hirase, "Design and control system of permanent-magnet synchronous motor for high torque and high efficiency operation," in *Proc. 22nd IEEE IAS Annu. Meet.*, Michigan, 1991, pp. 176–181.
[3] L. Hoang, R. Perret, and R. Feulliet. "Minimization of torque ripples in brushless DC motor drives," *IEEE Trans. Ind. Applicat.*, vol. IA-20, no. 4, pp. 803–813, 1987.
[4] C. V. Jones, *The Unified Theory of Elec. Machines*. London: Butterworth & Co. Ltd., 1967.
[5] T. M. Jahns, "Torque production in permanent-magnet synchronous motor drives with rectangular current excitation," *IEEE Trans. Ind. Applicat.*, vol. IA-20, no. 4, pp. 803–813, July/Aug. 1986.
[6] T. S. Low and C. Bi, "The use of finite elements and neural networks for the solution of inverse electromagnetic problems," *IEEE Trans. Magn.*, vol. 28, no. 5, pp. 101–103, Sept./Oct 1992.
[7] ——, "Neural-networks for the solution of constrained optimization of inverse electromagnetics problems," *Dig. INTERMAG'93*, pp. EQ-03, Apr., 1993.
[8] K. T. Chang, *Control Strategies for Permanent-Magnet Synchronous Motor Drives*, Ph.D. dissertation, National University of Singapore, 1994.

Teck-Seng Low (M'83–SM'93), for a photograph and biography, see p. 191 of the February 1996 issue of this Transactions.

Chao Bi (M'95), for a photograph and biography, see p. 191 of the February 1996 issue of this Transactions.



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