Neural Networks For The Solution of Constrained Optimization of Inverse Electromagnetic Problems

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Introduction

An inverse problem can generally be transformed into an optimization problem, and in most instances it is a constrained optimization problem. It is well known that the solution of constrained optimization problems are more difficult and many optimization methodologies which are effective in solving free optimization problems, cannot be applied to constrained optimization problems. This paper describes a methodology for constrained optimization as regard the solution of the inverse electromagnetic problem.

In paper [1], the authors introduced a method for solving the inverse electromagnetic problem. This method is known as the Finite Element-Neural Network method (FENN). In this method, the finite element method is used to produce the training patterns for analysing the optimized solution, and the neural network is used to form a mapping model for optimizing the parameters. It is found to be an effective method for solving the inverse problem. However the constrained conditons imposed in the problem impedes the optimizing process and results in low computation speed and problems in system stability.

Application of neural networks for the solution of constrained optimization

An example of a general constrained optimization problem is described by the Eq.(1).

$$\begin{cases} \min\limits_{x \in D_0} (f(X)) \\ D_0 = \{X | g_i(X) \le 0, i=1,2,\cdots,m\} \subset R^{n_x} \end{cases} \quad (1)$$

where the object function f(X) and the constrained conditions $g_i(X)$ $(i=1,2,\cdots,m)$ are the real functions in the space R^{nx} and $D_0 \subset R^{nx}$ is the allowed region. The points which belong to D_0 are called the allowed points. $g_i(X) \le 0$ $(i=1,2,\cdots,m)$ are the constrained conditions. In general, the *jth* constrained conditions $g_j(X)=g_j(x_1,x_2,\cdots,x_{nx}) \le 0$ can be rewritten as

$$x_j \ge p_j(x_1,x_2,\cdots,x_i,\cdots,x_{n_x}) \wedge x_j < q_j(x_1,x_2, \ ,x_i,\cdots,x_{n_x}) \quad (2)$$
$$(j=1,2,\cdots,m; i \ne j)$$

where, $p_j(X)$ and $q_j(X)$ are deduced from $g_j(X)$. In paper [1], the Forward Generating Neural Networks (FGNN)[2] was suggested for use in FENN. The architecture of the network is as shown in the Fig.1. As the constrained conditions of Eq.(2) have to be satisfied, this kind of network cannot be used directly for the solution of the optimization problem described by Eq.(1). The conditions shown in Eq.(2) can be changed to free conditions when an extra network is added before the input of the FGNN. This architecture is shown in Fig.2. This extra network is a multilayer network. The layer number is determined by the constrained conditions shown in Eq.(2). The squash functions in the neurons of the extra network are also determined by Eq.(2). This cascade network system takes the values of the free variables $[X]^*$ as inputs to the first network, and the outputs of the first network [X] are the constrained variables required. As an example, if α is to be constrained between 0-90°, then $\alpha=90°/(1+e^u)$. The constrained variable α is thus changed to a free variable u.

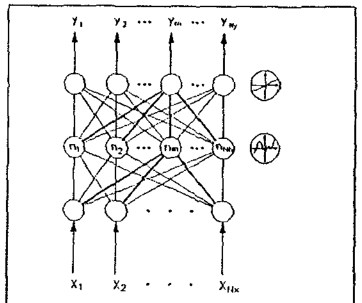

Fig.1 The architecture of the Forward Generating Neural Network

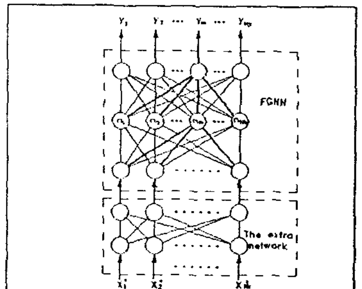

Fig.2 The neural network for solving the constrained optimization problems

In this paper, an example is shown where the slot shape of a BLDC rotor shell is optimized. In this example, the process of forming the extra network will be described for the conversion of the constrained variables to free variables.

It is found that the new network with the extra network is effective with the FENN for solution of constrained optimization problems.

References

[1] T. S. Low, Bi Chao, 'The use of finite elements and neural networks for the solution of inverse electromagnetic problems', Intermag 92, Digest paper, FD-05.

[2] T. S. Low, Bi Chao, 'Forward generating neural network: its architecture and training', Proc.IJCNN'91, Nov. 1991, pp404-409.